Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \dots + \binom{n}{r}a^{n-r}b^r + \dots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\dots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\Delta = \frac{1}{2}bc\sin A$$

1 The mass, m grams, of a substance present at time t days after first being measured is given by the formula $m = m_0 e^{-0.005t}$, where m_0 represents the initial mass of the substance. Find the value of t when the initial mass has been reduced by 20%. [3]

2 The function f is defined, for all values of x, by

$$f(x) = (2x - x^2)\, e^x.$$

Find the range of values of x for which f is a decreasing function. [4]

3 The gradient function of a curve is $2(p + 1)x + 2$, where p is a constant.

(i) State the condition for p if the curve has a maximum turning point. Justify your answer. [2]

(ii) Given that the tangent to the curve at $(1, -2)$ is parallel to $y + 2x - 5 = 0$. Find the value of p. [3]

4 The graph $y = \ln(3 - 2x)$ intersects the x-axis and y-axis at A and B respectively.

(i) Find the coordinates of A and of B. [2]

(ii) Explain why the graph will never meet the line $x = \dfrac{3}{2}$. [1]

(iii) Sketch the graph $y = \ln(3 - 2x)$. [2]

5 **(i)** Show that $\sin^4 x - \cos^4 x = -\cos 2x$. [3]

(ii) Hence, write down

(a) the range of $\sin^4 x - \cos^4 x + 1$, [2]

(b) the amplitude and period of $\sin^4 x - \cos^4 x + 1$. [2]

6 **(i)** Sketch the graph $y = \sqrt{x} - 1$. [2]

(ii) The line $y = x - 2$ intersects the graph $y = \sqrt{x} - 1$ at one point.

Show that the x-coordinate of this point is $\dfrac{3 + \sqrt{5}}{2}$. [5]

7 **(i)** Sketch the graph of $y = |x^2 - 4|$. [2]

(ii) Solve $|x^2 - 4| = -3x$. [4]

(iii) Determine the range of values of k such that $|x^2 - 4| = k$ has 4 solutions. [1]

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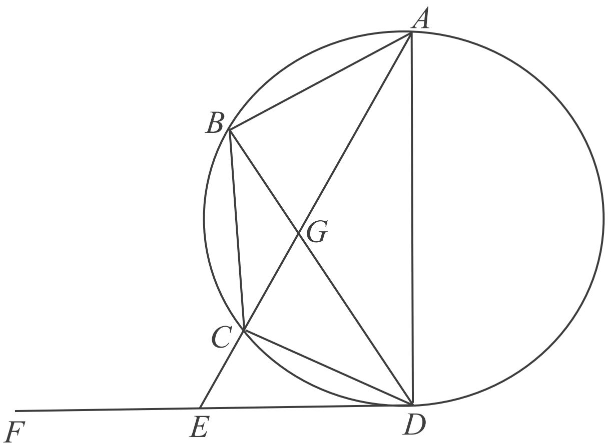

In the diagram, *ABCD* is a cyclic quadrilateral in which *EA* bisects angle *BAD* and *EA* cuts the circle at *C*. *DEF* is a tangent to the circle at *D* and *BGD* is a straight line.
Show that

(i) angle *CDE* = angle *CAB*, [2]

(ii) triangle *CDE* is similar to triangle *DAE*, [2]

(iii) $CE \times AE = DE^2$, [1]

(iv) *BC* = *CD*. [2]

9 **(i)** Show that $1 - 2\cot^2 A = \dfrac{5}{\sin A}$ can be expressed as $2\operatorname{cosec}^2 A + 5\operatorname{cosec} A - 3 = 0$. [3]

(ii) Hence solve the equation $1 - 2\cot^2 2\theta = \dfrac{5}{\sin 2\theta}$ for $0° < \theta < 360°$. [5]

[Turn over

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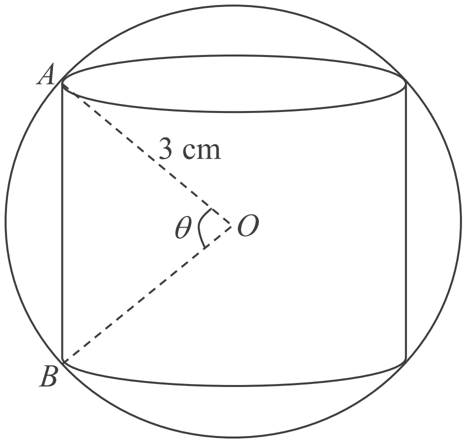

A solid right cylinder is removed from a solid sphere of radius 3 cm. O is the centre of the circle and angle $AOB = \theta°$.

(i) Show that the curved surface area, S cm^2, of the cylinder is given by

$$S = 18\pi \sin\theta.$$ [3]

(ii) Given that θ can vary, find the value of θ for which S has a stationary value. [3]

(iii) Determine whether this value of θ makes the curved surface area a maximum or a minimum. [2]

11 A particle P moves in a straight line so that its velocity, V m/s, from a fixed point O, is given by $V = t^2 - 5t + 6$, where t is the time in seconds after leaving O.

Find

(i) the values of t at which P is instantaneously at rest, [2]

(ii) the value of t for which the velocity is a minimum, [2]

(iii) the range of values of t for which the velocity of the particle is negative, [1]

(iv) the distance travelled by the particle in the third second. [4]

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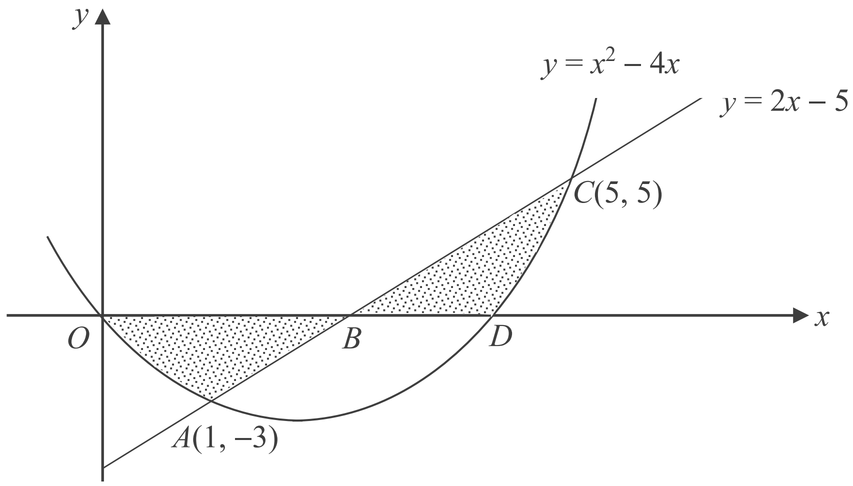

(i) The diagram shows part of the curve of $y = x^2 - 4x$. The curve meets the x-axis at D and at the origin, O. The line $y = 2x - 5$ meets the curve at the points $A(1, -3)$ and $C(5, 5)$. B is the point of intersection of the line $y = 2x - 5$ and the x-axis. Find the total area of the shaded regions. [6]

(ii) A point P moves along the curve in such a way that the x-coordinate of P increases at a constant rate of $\frac{5}{6}$ units per second. Find the x-coordinate of P at the instant when the y-coordinate is decreasing at $\frac{5}{6}$ units per second. [4]

End of Paper