Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$\left(a+b\right)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n ,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$

1 **(a)** Differentiate $\ln\left(\frac{x+1}{2-x}\right)$ with respect to x. [2]

(b) A curve is such that $\frac{dy}{dx} = -\frac{1}{2}e^{2-x} - 1$, where $x < 1$.

(i) Determine whether the curve has a stationary point. [2]

(ii) Given that the curve passes through the point $\left(0, \frac{1}{2}e^2\right)$, find the equation of the curve. [3]

2 **(i)** Differentiate $x\cos 3x$ with respect to x. [3]

(ii) Using your answer to part **(i)**, find $\int x\sin 3x \, dx$ and hence show that $\int_0^{\frac{\pi}{3}} x\sin 3x \, dx = \frac{\pi}{9}$. [5]

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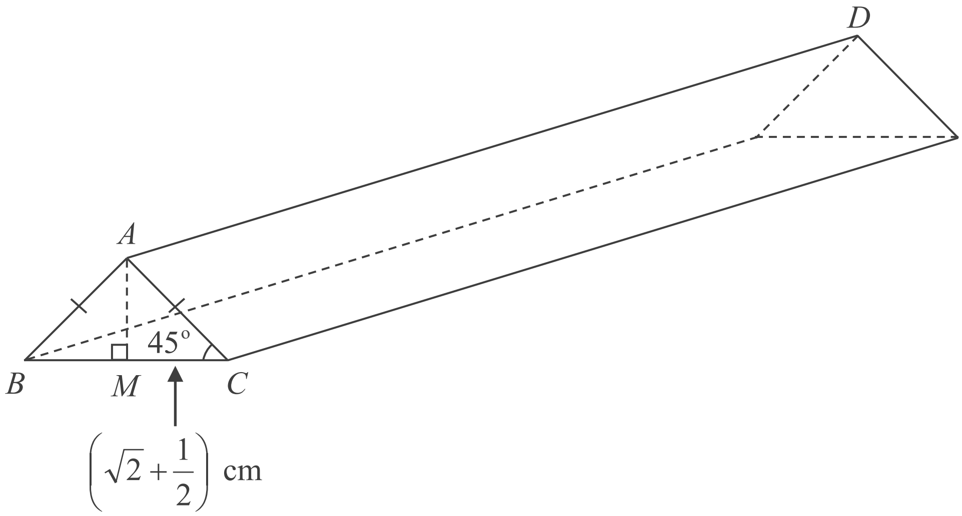

The diagram shows a chocolate bar in the form of a triangular prism and the cross-section of the chocolate bar is an isosceles triangle. $MC = \left(\sqrt{2}+\frac{1}{2}\right)$ cm and angle $ACB = 45°$.

(i) Find the exact length of AC. [3]

(ii) Find an expression for the area of the cross-section of the chocolate bar in the form $\left(a + b\sqrt{2}\right)$ cm^2, where a and b are rational numbers. [3]

(iii) Given that the volume of the chocolate bar is $\left(25 + 22\sqrt{2}\right)$ cm^3, find the length of AD in the form $\left(c + d\sqrt{2}\right)$ cm, where c and d are integers. [3]

4 **(a)** Given that $(2+ax)^5(1+3x-2x^2)=32-144x+bx^2+\ldots$, find the value of a and of b. [4]

(b) Given that the coefficients of x^{11} and x^{12} in the expansion of $(2+kx)^{19}$ are in the ratio 3 : 5, find the value of k. [4]

5 The curve $y=x^3+\dfrac{3}{x^2}$ passes through the point $P(1, 4)$. The tangent to the curve at P meets the x-axis at A and the normal to the curve at P meets the x-axis at B.

(i) Find the coordinates of A and of B. [7]

(ii) Find the area of triangle APB. [2]

6 **(i)** Find the coordinates of the stationary points of the curve $y=\dfrac{x^2}{2x+1}$. [5]

(ii) Determine the nature of each of the stationary points. [4]

7 A circle C, which passes through the origin, meets the x-axis and y-axis at (1, 0) and (0, 2) respectively.

(i) Find the equation of C. [3]

The line $y=x+k$, where k is a constant, is a tangent to the circle C.

(ii) Find the possible values of k, leaving your answers in the simplest surd form. [6]

8 The roots of the quadratic equation $2x^2-2x+5=0$ are α and β.

(i) Find the value of $\alpha^2+\beta^2$. [3]

(ii) Using your answer in part **(i)**, find the value of $\alpha^3+\beta^3$ and of $\alpha^3\beta^3$. [3]

(iii) Find a quadratic equation whose roots are $\dfrac{1}{\alpha^3}$ and $\dfrac{1}{\beta^3}$. [4]

[Turn over

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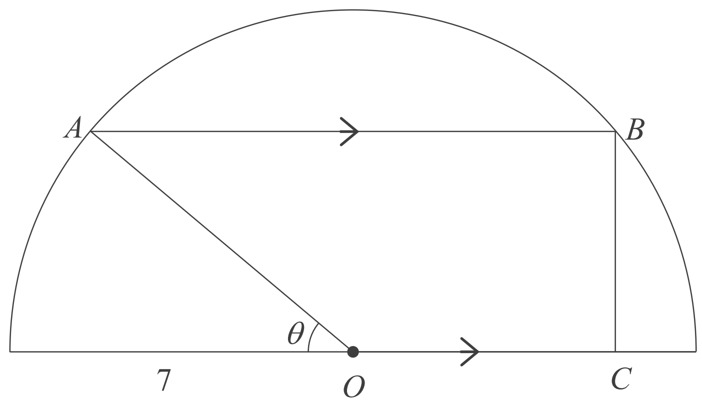

The diagram shows a trapezium *OABC* inscribed in a semicircle with centre *O*, and radius 7 cm. *OA* makes an angle θ with the diameter. *BC* is perpendicular to both *AB* and *CO*.

(i) State the property that shows that *AB* is twice the length of *OC*. [1]

(ii) Show that *P* cm, the perimeter of the trapezium, can be expressed in the form $m+n\cos\theta+q\sin\theta$, where *m*, *n* and *q* are constants to be found. [3]

(iii) Express *P* in the form $m+R\cos(\theta-\alpha)$, where $R>0$ and α is an acute angle. [3]

(iv) Hence, find the maximum value of *P* and the corresponding value of θ at which this occurs. [3]

10 **(a)** The expression $f(x)=x^3+ax^2+bx+c$ leaves the same remainder, *R*, when it is divided by $x+2$ and when it is divided by $x-2$.

(i) Find the value of *b*. [2]

f(*x*) also leaves the same remainder, *R*, when divided by $x-1$.

(ii) Find the value of *a*. [2]

f(*x*) leaves a remainder of 4 when divided by $x-3$.

(iii) Find the value of *c*. [1]

(b) Given that $4x^4-12x^3-b^2x^2-7bx-2$ is exactly divisible by $2x+b$,

(i) show that $3b^3+7b^2-4=0$, [2]

(ii) find the possible values of *b*. [4]

11 The table shows experimental values of the variables x and y which are related by the equation $y = Ab^x$, where A and b are constants.

x	2	4	6	8	10
y	9.8	19.4	37.4	74.0	144.4

(i) Using suitable variables, draw, on graph paper, a straight line graph. [4]

(ii) Use your graph to estimate the value of A and of b. [3]

(iii) On the same diagram, draw the straight line representing $y = 2^x$ and hence find the value of x for which $A = \left(\frac{2}{b}\right)^x$. [3]

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