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ANDERSON SECONDARY SCHOOL
Preliminary Examination 2017
Secondary Four Express & Five Normal

CANDIDATE NAME:

CLASS: / INDEX NUMBER:

ADDITIONAL MATHEMATICS 4047/01

Paper 1

25 August 2017

2 hours

0800 – 1000h

Additional Materials: Writing paper

READ THESE INSTRUCTIONS FIRST

Write your class, index number and name on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid/tape.

Answer **all** the questions.
Write your answers on the separate Answer Paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

The total of the marks for this paper is 80.

This document consists of **6** printed pages.

Setter: Ms Lee Siew Lin

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

Binomial Theorem

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$.

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1.$$
$$\sec^2 A = 1 + \tan^2 A.$$
$$\operatorname{cosec}^2 A = 1 + \cot^2 A.$$
$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$
$$\sin 2A = 2 \sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$
$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}.$$
$$a^2 = b^2 + c^2 - 2bc \cos A.$$
$$\Delta = \frac{1}{2}bc \sin A.$$

1 A curve is such that $\frac{dy}{dx} = 5 - \sqrt{x-3}$ and A (7, 15) is a point on the curve.

(i) Find the equation of the curve. [2]

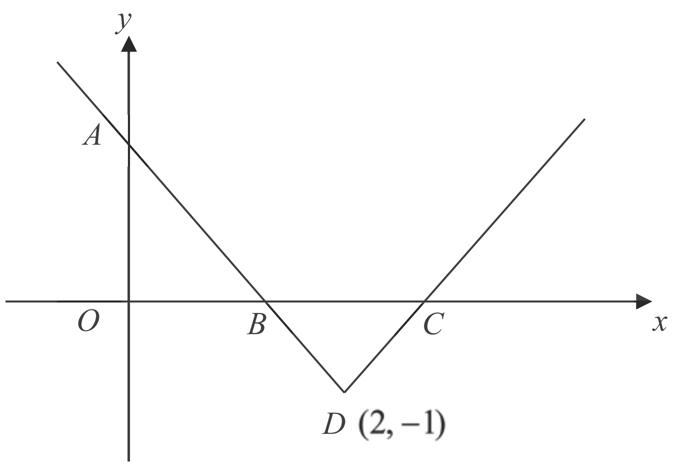
(ii) Find the equation of the normal to the curve at A. [2]

2 The equation of a curve is given by $y = x^2 + 2ax + 2a - 3$, where a is a constant. Show that, for all values of a, the curve intersects the x-axis at two distinct points. [4]

3 The diagram shows the graph of $y = |p - 2x| + q$, where p and q are integers. A is the point where the graph intersects the y-axis, and B and C are the points where it intersects the x-axis. Point $D(2, -1)$ is the vertex of the graph.

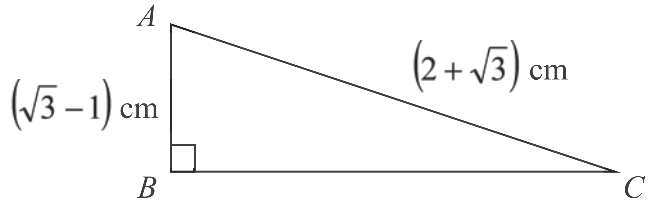
(i) Find the value of p and of q. [2]

(ii) Hence find the coordinates of A, B and C. [4]



4 **(i)** Express $\left(\frac{1-\sqrt{3}}{2+\sqrt{3}}\right)^2$ in the form $a + b\sqrt{3}$ where a and b are integers. [3]

(ii) The diagram shows a triangle ABC where angle $ABC = 90°$, $AB = (\sqrt{3} - 1)$cm and $AC = (2 + \sqrt{3})$cm . Using your answer to **(i)**, find the exact value of $\cos^2(A\hat{C}B)$ without using a calculator. [3]



5 It is given that point A $(8, 11)$ lies on the line l with equation $y = 2x - 5$, and P is the point (1, 2).

If B is the point on l such that PBA is a right-angled triangle, find

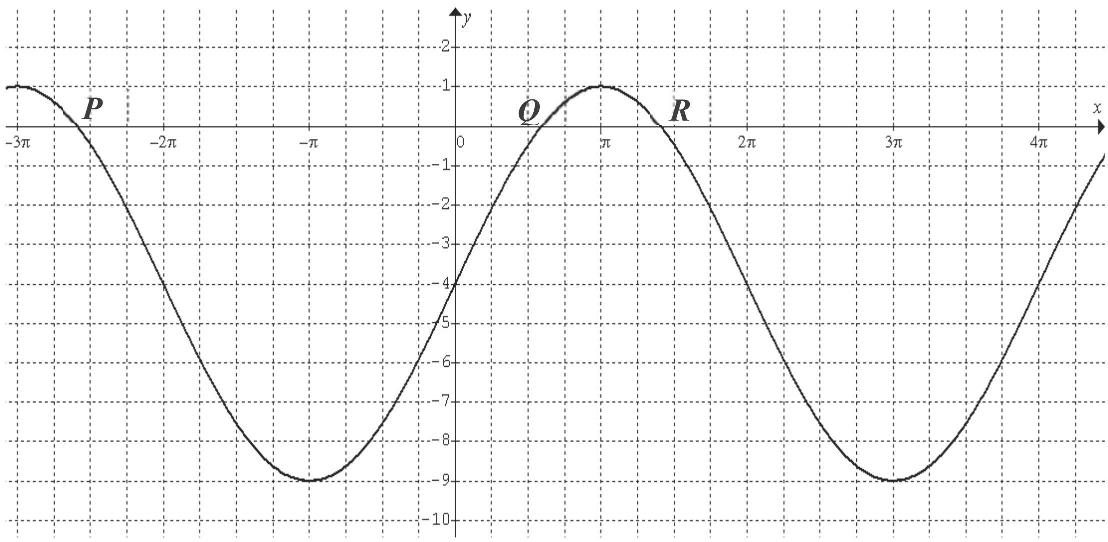
(i) the coordinates of B, [5]

(ii) the area of triangle PBA. [2]

6 **(i)** Prove that $(\sin 2y + 2)(\sin y - \cos y) = 2\cos^3 y\,(\tan^3 y - 1)$. [4]

(ii) Hence find the acute angle y, in degrees, such that $(\sin 2y + 2)(\sin y - \cos y) = 2\cos^3 y$. [2]

7 The figure shows part of the graph of $y = a\sin(bx) + c$. Points P, Q and R on the graph lie on the x axis.



(i) Find the value of each of the constants a, b and c. [3]

(ii) Hence find the coordinates of P, Q and R. [4]

8 A curve has the equation $y = 4-(3x-1)^2$.

(i) Explain why the highest point on the curve has coordinates $\left(\frac{1}{3}, 4\right)$. [1]

(ii) Find the coordinates of the points at which the curve intersects the x-axis. [2]

(iii) Sketch the graph of $y = \left|4-(3x-1)^2\right|$. [2]

(iv) The equation $\left|4-(3x-1)^2\right| = mx+4$ has 4 distinct solutions. Using your graph, determine the range of values of m. [2]

9 **(i)** Show that $\frac{\mathrm{d}}{\mathrm{d}x}\left(x\cos^2 3x\right) = \cos^2 3x - 3x\sin 6x$. [3]

(ii) Hence integrate $x\sin 6x$ with respect to x. [4]

10 **(i)** Sketch the parabola $y^2 = 2x$. [2]

(ii) The curve $y^2 = 2x$ intersects the straight line $y = 3x-1$ at the points A and B. Find the coordinates of the midpoint of AB. [6]

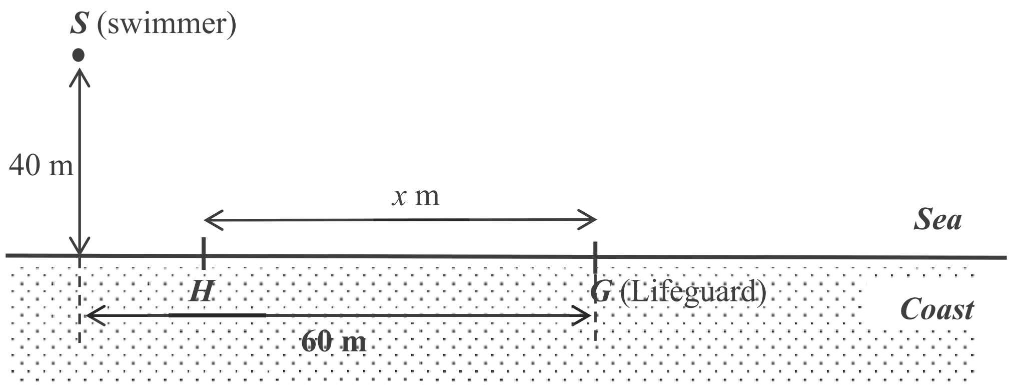
11 **(i)** Express $\frac{4x^3+45x^2+126x+16}{(2x-1)(x+5)^2}$ in partial fractions. [5]

(ii) Hence show that

$$\int_1^2 \frac{4x^3+45x^2+126x+16}{(2x-1)(x+5)^2}\,\mathrm{d}x = \frac{83}{42}+\frac{\ln 27}{2}+\ln\left(\frac{49}{36}\right).$$ [4]

12 A lifeguard at a beach resort is stationed at point G along the coastline, as shown in the diagram below. When he detects a swimmer who needs help at a point S, he would run along the coastline over a distance of x m to a point H, and then swim in a straight line, HS, towards the swimmer. The lifeguard runs at a speed of 4 m/s and swims at a speed of 2 m/s.

A swimmer in distress is detected at a position that is 40 m away from the coastline, and the foot of perpendicular from the swimmer to the coastline is at a distance of 60 m away from the lifeguard.



(i) Show that the time taken by the lifeguard to swim from H to S is $\dfrac{\sqrt{1600+(60-x)^2}}{2}$ seconds. [2]

(ii) Find, in terms of x, the total time T taken by the lifeguard to reach the swimmer. [1]

(iii) Obtain an expression for $\dfrac{dT}{dx}$. [2]

(iv) Find the value of x such that the lifeguard would be able to reach the swimmer in the shortest possible time. [4]

END OF PAPER