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ANDERSON SECONDARY SCHOOL
Preliminary Examination 2017
Secondary Four Express & Five Normal

CANDIDATE NAME:

CLASS: /

INDEX NUMBER:

ADDITIONAL MATHEMATICS 4047/02

Paper 2

28 August 2017

2 hours 30 minutes

1000h – 1230h

Additional Materials: Writing paper
Graph paper (1 sheet)

READ THESE INSTRUCTIONS FIRST

Write your class, index number and name on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid/tape.

Answer **all** the questions.
Write your answers on the separate Answer Paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is 100.

This document consists of **6** printed pages.

Setter: Mr Wong Teck Hock

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ \ldots\ (n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} ab \sin C$$

1 **(i)** Given that the coefficient of x^2 in the expansion of $(1-2x)^2(1+px)^7$ is 32, find the possible values of the constant p. [4]

(ii) Hence find the coefficient of x^2 in the expansion of $(1-x)^2\left(1+\frac{px}{2}\right)^7$. [2]

2 **(i)** Using $\sin 3x \equiv \sin(2x+x)$, show that $\sin 3x$ may be expressed as $3\sin x - 4\sin^3 x$. [3]

(ii) Hence solve the equation $\sin 3x = 2\sin x$ for $0 < x < 2\pi$. [5]

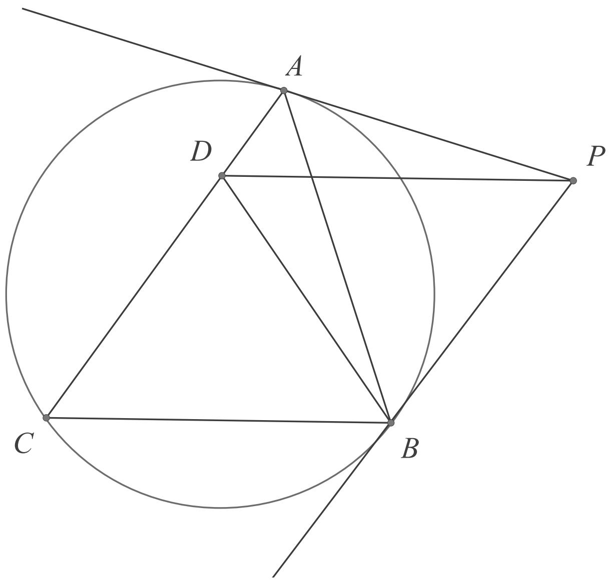
3 The roots of a quadratic equation $x^2 = 12x - 4$ are α^2 and β^2 where $\alpha < 0, \beta < 0$ and $\alpha < \beta$. Find, without calculating the value of α and β,

(i) the value of $\alpha\beta$, [2]

(ii) the value $\alpha + \beta$, [3]

(iii) a quadratic equation with roots $\alpha + \beta$ and $\alpha - \beta$, in the form $(x+a)(x+b) = 0$ where a and b are real numbers. [3]

4



The diagram shows a circle passing through the vertices of a triangle ABC. The tangents to the circle at A and B intersect at the point P. The point D lies on AC such that PD is parallel to BC. Prove that

(i) angle $ADP =$ angle ABP, [2]

(ii) A, D, B and P lie on a circle, [1]

(iii) $DB = DC$. [4]

5 It is given that $y = \dfrac{1+2x}{e^{3x}}$.

(i) Obtain an expression for $\dfrac{dy}{dx}$ in the form $\dfrac{ax+b}{e^{3x}}$, where a and b are integers. [2]

(ii) Determine the range of values of x for which y is decreasing. [3]

The values of x and y are such that, when $x=1$, y is increasing at a rate of $\dfrac{1}{4}$ units per second.

(iii) Find the exact rate of change of x when $x=1$. [3]

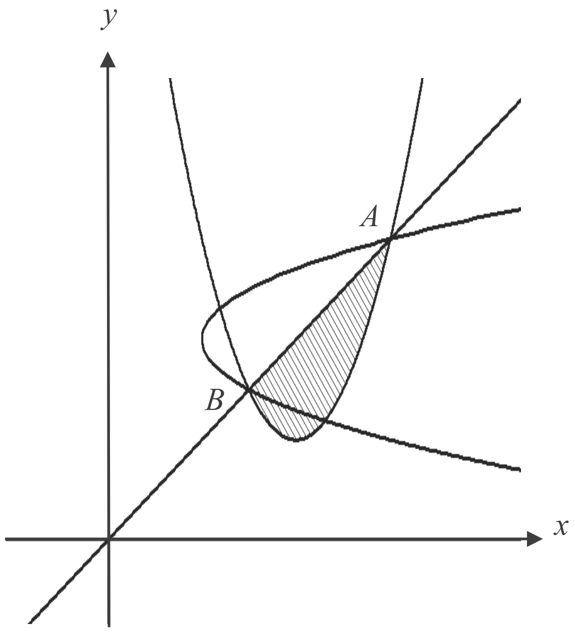
6 It is given that $f(x) = 2x^3 + x^2 - 8x - 4$.

(i) Factorise $f(x)$, showing your working clearly. [3]

(ii) Show that the equation $f(x) + 10x + 5 = 0$ has only one real root and state its value. [3]

(iii) Find the range of value of the constant k for which the graph of $y = f(x) + kx$ has two stationary points. [4]

7



The diagram shows part of the graphs of $y = 3x^2 - 8x + 6$, $x = 3y^2 - 8y + 6$ and the line $y = x$, intersecting at the points A and B.

(i) Find the x-coordinates of A and B. [3]

(ii) Find the area of the shaded region bounded by the curve $y = 3x^2 - 8x + 6$ and the line $y = x$. [3]

(iii) Deduce the area of the region bounded by the curve $x = 3y^2 - 8y + 6$ and the line $y = x$. [2]

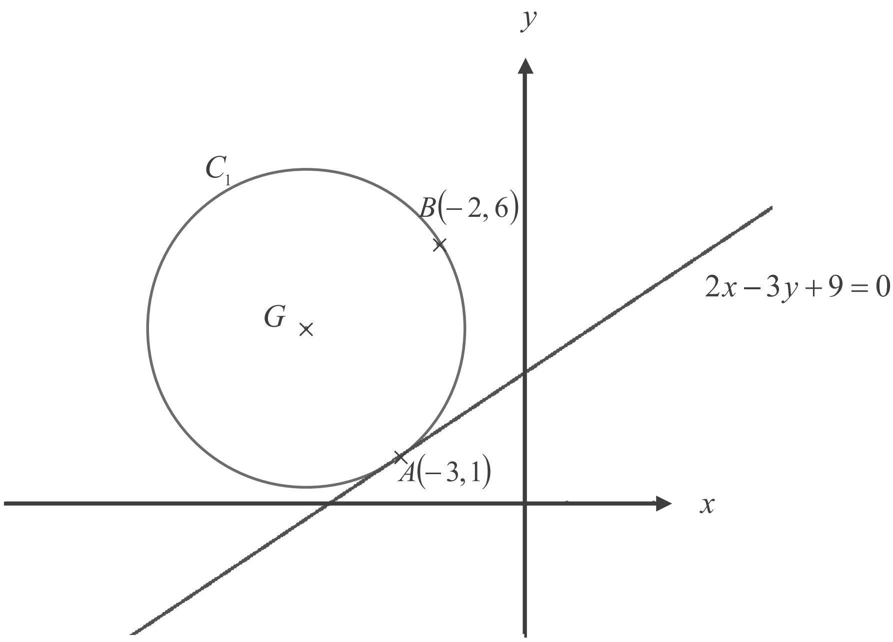
8 A point P moves along a straight line such that its displacement, x cm, t seconds after leaving a fixed point, is given by $x = 5\cos 2t - 6\sin t$. Find

(i) the exact velocity of P when $t = \dfrac{\pi}{6}$, [3]

(ii) the value of t between 0 and π for which P is instantaneously at rest, [4]

(iii) the distance travelled by P in the first 2 seconds. [4]

9



In the diagram, the circle C_1 with centre G passes through the point $B(-2, 6)$ and touches the line $2x - 3y + 9 = 0$ at the point $A(-3, 1)$.

(i) Find the equation of the perpendicular bisector of AB. [3]

(ii) Show that G is $(-5, 4)$. [4]

(iii) Find the equation of the circle C_1. [2]

A second circle, C_2 with centre H is a reflection of the circle C_1 along the line $2x - 3y + 9 = 0$. Find

(iv) the coordinates of H, [2]

(v) the equation of circle C_2. [1]

10 **(a)** **(i)** Explain why the equation $\sqrt{3-e^x}+1-ke^x=0$ has no solution if $k<0$. [3]

(ii) Solve the equation $3-\sqrt{3-e^x}=e^x$. [3]

(b) In a chemical reaction, the mass, x kg, of a certain substance produced after t hours is given by the equation $\ln\left(\frac{ax}{1-x}\right)=bt$ where a and b are positive constants to be determined.

(i) Given that the initial mass of the substance is $\frac{1}{5}$ kg and $x=\frac{1}{3}$ when $t=1$, determine the exact value of a and of b. [4]

(ii) Express x in terms of t. [2]

11 Mr Lee bought a car on 1st January 2013. The market value of the car decreases each year from 2013 to 2017. He estimated that the value $\$V$ of his car t years after 1st January 2013 can be modelled by the equation

$$V=V_0a^t,$$

where V_0 and a are constants. The table below gives values of V and t.

t	1	2	3	4
V	81 000	72 900	65 600	59 000

(i) Plot a suitable straight line graph for $4.5<\lg V<5.0$ and show that the model is valid from 2013 to 2017. [4]

(ii) Estimate the value of V_0 and a. Hence deduce the price Mr Lee paid for the car on 1 January 2013. [5]

(iii) Assuming that the model is still appropriate, estimate the market value of the car on 1st January 2018. [1]

End of Paper