3

Answer all the questions.

Given that $\cos A = \frac{3}{5}$ and $\tan B = -\frac{5}{12}$ where A and B lies in the same quadrant, find the exact value of

(i)
$$tan A$$
, [1]

(ii)
$$\sin(B)$$
, [1]

(iii)
$$\operatorname{cosec}(180^{\circ} - B)$$
. [2]

- Solve the equation $6\sin x + 4\tan x = 2\sec x + 3$ for $-180^{\circ} \le x \le 180^{\circ}$. [4]
- The curve $y^2 = 72x$ intersects the curve $y = 3x^2$ at A and B. Find the equation of the line joining A and B. [5]
- Find the range of values of m for which the curve $y = (m-3)x^2 + 3x + (m+1)$ lies entirely above the x-axis. [5]
- 5 Express $\frac{4x^3 2x^2 13x + 13}{2x^2 2}$ in its partial fractions. [5]
- A prism has a regular hexagonal cross-section with sides of length $(2+\sqrt{3})$ cm. Given that the volume of the prism is $\frac{3}{2}(17\sqrt{3}+30)$ cm³, find the height of prism, leaving your answer in the form of $(a\sqrt{3}+b)$ cm where a and b are integers. [6]
- 7 (i) Sketch, on the same set of axes, the graph of $y = 2\cos 3x + i$ and $y = \tan x$ for $0 \le x \le \pi$. [4]
 - (ii) Hence, state the number of solutions to the following equations.

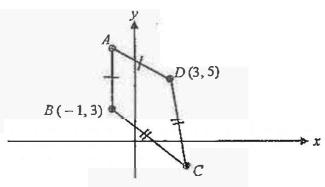
(a)
$$\tan x = 2\cos 3x + 1$$
, [1]

(b)
$$|2\cos 3x + 1| = \tan x$$
. [1]

Show that the expression |4x-6|+|9-6x| can be simplified to the form k|2x-3| where k is an integer. [2]

(ii) Sketch the graph of
$$y = |4x - 6| + |9 - 6x| - 2$$
. [3]

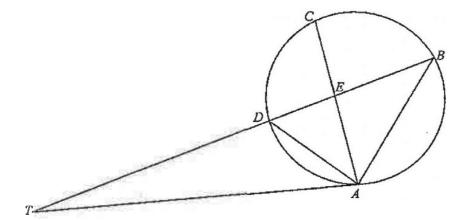
- (iii) Hence, determine the range of values of m for which the line y = mx 5 cuts the graph y = |4x 6| + |9 6x| 2 at 2 points. [2]
- 9 (i) By expressing $y = \ln \sqrt{\frac{x^2 + 6x}{3x^2 + 16x 12}}^3$ in the form of $y = a [\ln f(x) \ln g(x)]$, where a is a constant and f(x) and g(x) are linear functions, find the first derivative of y. Express your answer as a single fraction. [5]
 - (ii) Hence, explain why the curve $y = \ln \sqrt{\frac{x^2 + 6x}{3x^2 + 16x 12}}^3$ has no turning points for $x > \frac{3}{2}$.
- 10 (a) Show that $\frac{d}{dx} \left[\cot^2 \left(\frac{\pi}{2} 2x \right) \sin 2x \right] = \frac{2 \sin^2 2x (2 + \cos^2 2x)}{\cos^3 2x}$ [4]
 - (b) Find the exact value of $\int_0^{\frac{\pi}{3}} (5 \tan^2 2x + 3) dx$. [3]
- Find the coordinates of the stationary point(s) of the curve $y = x^4 3x^3 + 1$ and determine the nature of the stationary point(s). [8]
- In the diagram below, ABCD is a kite where the coordinates of B is (-1,3) and the coordinates of D is (3,5).



- (i) Given that A lies vertically above B, find the coordinates of A.
- (ii) Given that the ratio of the area of triangle ABD: area of triangle BCD = 2:3, find the coordinates of C. [4]

[4]

In the diagram below, A, B, C and D are points on the circle and AT is a tangent to the circle. BDT is a straight line such that BD: BT = 2:5. AC meets BD produced at E such that BE: ED = 3:2.



(i) Prove that triangle ATD is similar to triangle BTA.

(ii) Prove that $2AT^2 = S(TB \times BE)$. [5]

[3]

End of Paper 1