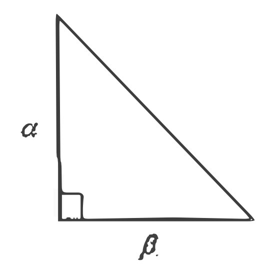
Answer all the questions.

1 (a) A function is given by $y=\frac{e^{2x}}{2x-3}$. Show that the y is an increasing function for $x>2$. [4]

(b) It is given that $y=(2x+5)(x-4)^2$, where x is positive. Find the exact value of x when the rate of increase of y is thrice the rate of decrease of x. [4]

2 (a) The lengths, α and β in cm, of the two shorter sides of a right-angled triangle, shown in the diagram below, are the roots of the equation $2x^2-15x+26=0$. **Without solving the equation**, find the area and perimeter of this triangle. [4]

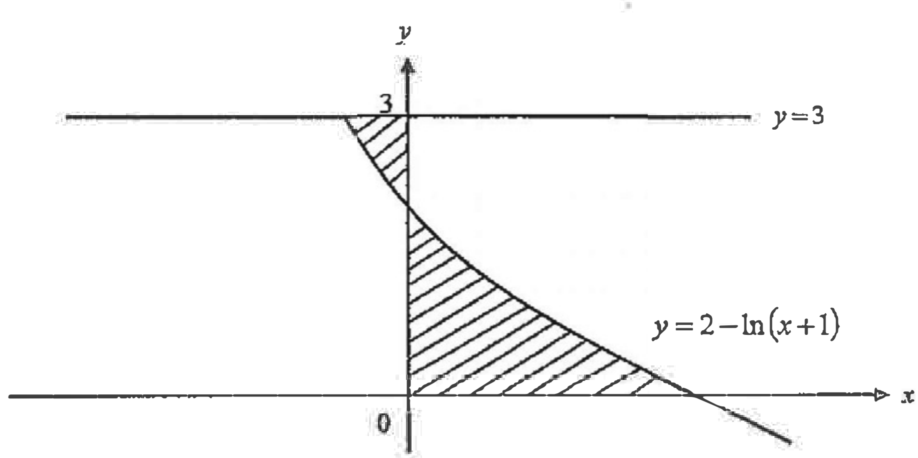


(b) The roots of the equation $3x^2+kx+96=0$ are both positive and one root is twice the other root. Calculate the value of each root and find k. [4]

3 The diagram shows part of the curve $y=2-\ln(x+1)$.

(i) Find the coordinates of the point where the curve cuts the y-axis. [1]

(ii) Calculate the area of the shaded region bounded by the curve $y=2-\ln(x+1)$, the axes an $y=3$. [7]



4 The polynomial $P(x)=2x^3+ax^2+bx+21$, where a and b are constants. It is given that $P(x)$ leaves a remainder of 35 when divided by $2x-1$ and has a factor of $x+3$.

(i) Find the value of a and of b. [5]

(ii) Explain why the equation $P(x)=0$ has only 1 real root. State the value of this root. [4]

5 (a) The equation of a curve is $y=\frac{1}{\sqrt{ax+1}}$, where a is a constant. The gradient of the normal to the curve where the curve passes through the y-axis is 1. Find the value of a. [3]

(b) Given that the first two non-zero terms of the expansion of $(1-kx)\left(1+\frac{x}{3}\right)^n$ in ascending power of x are 1 and $-\frac{5x^2}{3}$, where n is a positive integer, find the value of k and of n. [7]

6 A rectangular plot of land is used to grow watermelon. The area of the land occupied by the watermelon is y m² has sides of length x m and $(Ax+B)$ m, where A and B are constants and x and y are variables. Values of x and y are given in the table below.

x	20	40	60	80	100	120
y	960	2320	4080	6240	8800	11760

(i) Plot $\frac{y}{x}$ against x and draw a straight line graph. [3]

(ii) Use your graph to estimate value of A and of B. [4]

(iii) On the same diagram, draw the straight line representing the equation $y=2x^2$ and explain the significance of the value of x given by the point of intersection of the two lines. [3]

7 (i) The equation of circle C_1 is given by $x^2+y^2-2kx+2y+1=0$, where k is a positive constant. Given that C_1 has a radius of 2 units, find the value of k. [3]

(ii) The centre of a circle C_2 lies on the line $y=2x+2$. Given that C_2 passes through the points $(3, 2)$ and $(0,-1)$, find the equation of C_2. [5]

(iii) Calculate the shortest distance from the centre of C_1 to the circumference of C_2. [3]

8 A particle travelling in a straight line passes a fixed point O with a velocity of 48 m/s. Its acceleration a m/s^2 is given by $a = 12t - 36$, where t is time in seconds after passing O.

(i) Find minimum velocity of the particle. [3]

(ii) Find the displacement of the particle from O when it is first at rest. [3]

(iii) Find the total distance travelled by the particle during the first 5 seconds. [3]

(iv) Show that the particle will never return to its starting point. [3]

9 (a) Given that $2^{2x+2} \times 5^{x-1} = 8^x \times 5^{2x}$, find the value of 10^x without using a calculator. [3]

(b) Solve the equation $\log_8[\log_4(5x-9)] = \log_{27} 3$. [3]

(c) Farmers use pesticide on a vegetable farm for t days and the number of worms, W, on the farm is given by $W = 3600(3 + e^{-0.18kt})$, where k is a constant. The number of worms has reduced by 10% after 5 days.

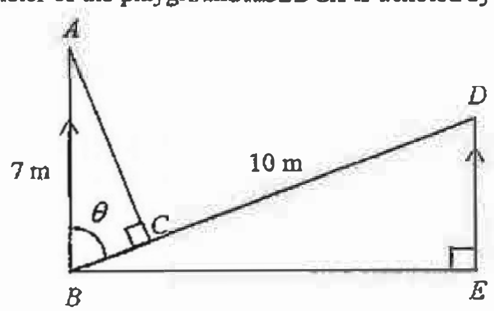
(i) Find the initial number of worms on the farm before the pesticide is used. [1]

(ii) Find the value of k. [3]

(iii) Explain whether the number of worms left on the farm would eventually fall below 10000. [2]

10 (a) Prove the identity $\dfrac{\sin 2A + \cos 2A + 1}{\sin 2A + \cos 2A - 1} = \dfrac{1 + \cot A}{1 - \tan A}$. [4]

(b) The diagram shows a playground made up of 2 right-angled triangles, ABC and BDE. It is given that AB and DE are parallel and $AB = 7$ m, $BD = 10$ m and $\angle ABC = \theta$, where θ is acute. The perimeter of the playground $ABEDCA$ is denoted by P.



(i) Show that $P = 17 + 17\sin\theta + 3\cos\theta$. [3]

(ii) Express P in the form $a + b\sin(\theta + \alpha)$, where $b > 0$ and $0 < \alpha < \dfrac{\pi}{2}$. [3]

(iii) State the maximum value of P and the corresponding value of θ. [2]

End of Paper 2