2016 AM 4E Prelim Paper 1 Marking Scheme

	Solutions:
1	$m = m_0 e^{-0.005t}$ $*0.8m_0 = m_0 e^{-0.005\,t}$ $0.8 = e^{-0.005\,t}$ $-0.005t = \ln 0.8$ $t = \dfrac{\ln 0.8}{-0.005} = 44.6$
2	$f(x) = (2x - x^2)\,e^x$ $f'(x) = (2x - x^2)\,e^x + e^x\,(2 - 2x)$ $= (2x - x^2 + 2 - 2x)e^x$ $= e^x(2 - x^2)$ For decreasing function, $f'(x) < 0$ $e^x(2 - x^2) < 0$ since $e^x > 0$, $(2 - x^2) < 0$ ➔ $(\sqrt{2} - x)(\sqrt{2} + x) < 0$ $\therefore\ x < -\sqrt{2},\quad x > \sqrt{2}$
3	$\dfrac{dy}{dx} = 2(p + 1)x + 2$ $y = (p + 1)x^2 + 2x + c$
(i)	For curve completely below x-axis, Coeff of $x^2 < 0$, → $p + 1 < 0$, $p < -1$
(ii)	$y + 2x - 5 = 0.$ $y = 5 - 2x$ gradient $= -2$ when $x = 1$ $2(p + 1)(1) + 2 = -2$ $2p + 2 = -4$ $p = -3$

Solutions:

4 $y = \ln(3-2x)$

(i) For $y = 0$, $\ln(3-2x) = 0$

$$3 - 2x = 1 \rightarrow x = 1$$

$\therefore A = (1, 0)$

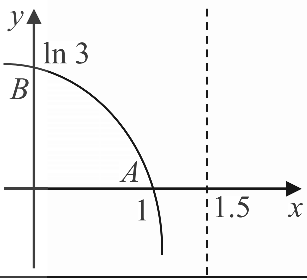
For $x = 0$, $y = \ln(3-0) = \ln 3$ (or 1.10)

$\therefore B = (0, \ln 3)$

For y to be defined, $(3 - 2x) > 0$

$$\rightarrow x < \frac{3}{2}$$

The line $x = \frac{3}{2}$ is the asymptote of the graph and hence the graph will never meet the line.



5 $\sin^4 x - \cos^4 x$

(i) $= (\sin^2 x - \cos^2 x)(\sin^2 x + \cos^2 x)$

$= -(\cos^2 x - \sin^2 x)(1)$

$= -\cos 2x$.

(ii) Let $y = \sin^4 x - \cos^4 x + 1 = 1 - \cos 2x$

(a) $\therefore$ range is $0 \le y \le 2$.

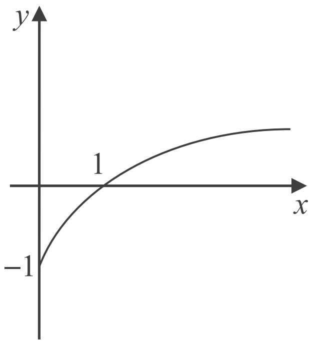
(b) Amplitude = 1

Period = 180° or π

Solutions:

6

(i)



(ii)

$x-2=\sqrt{x}-1$

$x-1=\sqrt{x}$

$(x-1)^2=x$

$x^2-3x+1=0$

$x=\dfrac{3\pm\sqrt{(-3)^2-4(1)(1)}}{2}$

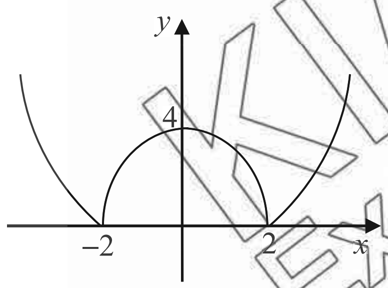
$=\dfrac{3\pm\sqrt{5}}{2}$

When $x=\dfrac{3-\sqrt{5}}{2}$, $x-1\neq\sqrt{x}$

$\therefore\ x=\dfrac{3+\sqrt{5}}{2}$ (shown)

7

(i)



(ii)

$|x^2-4|=-3x$

$x^2-4=3x$ or $x^2-4=-3x$

$x^2-3x-4=0$ $\quad$ $x^2+3x-4=0$

$(x-4)(x+1)=0$ $\quad$ $(x+4)(x-1)=0$

$x=4$ or $x=-1$ $\quad$ $x=-4$ or $x=1$

(rej) $\quad$ (rej)

Since $|x^2-4|\geq 0$ $\therefore\ x=-1$ or -4

(iii)

$0<k<4$

	Solutions:
8 (i)	$\angle CDE = \angle CAD$ (alt seg Thm) $= \angle CAB$ (EA bisects $\angle BAD$) (shown)
(ii)	$\angle CED = \angle DEA$ (common angle) $\angle CDE = \angle DAE$ (alt seg Thm) $\therefore\ \Delta\, CDE$ is similar to $\Delta\, DAE$
(iii)	$\frac{CE}{DE} = \frac{DE}{AE}$ $\therefore\ CE \times AE = DE^2$ (shown)
(iv)	$\angle CBD = \angle CAD$ ($\angle$s in same segment) $= \angle CAB$ (EA bisects $\angle BAD$) $= \angle CDB$($\angle$s in same segment) $\therefore \Delta\, BCD$ is isos (same base angles) Hence $BC = CD$. (shown)
9 (i)	$1 - 2\cot^2 A = \frac{5}{\sin A}$ $1 - 2(\text{*cosec}^2 A - 1) = \text{*}5 \text{ cosec } A$ $2\text{cosec}^2 A + 3 = 5 \text{ cosec } A$ $2\text{cosec}^2 A - 5\text{cosec } A + 3 = 0$ (shown)
(ii)	$2\text{cosec}^2 2\theta - 5 \text{ cosec } 2\theta + 3 = 0$ $(2\text{cosec } 2\theta - 1)(\text{cosec } 2\theta + 3) = 0$ $\text{cosec } 2\theta = 0.5$ or $\text{cosec } 2\theta = -3$ $\sin 2\theta = 2$ or $\sin 2\theta = -\frac{1}{3}$* *(no solution) or $\alpha = 19.47°$ $2\theta = 199.47°, 340.53°, 559.47°, 700.53°$ $\theta = \underbrace{99.7°, 170.3°}_{*}, \underbrace{279.7°, 350.3°}_{*}$
10 (i)	Radius of cylinder = 3cos Height = $AB = 6\sin\frac{\theta}{2}$ $S = 2\pi rh$ $= 2\pi\,(3\cos\frac{\theta}{2})(6\sin\frac{\theta}{2})$ $= 18\pi(2\sin\frac{\theta}{2}\cos\frac{\theta}{2})$ $= 18\pi\sin\theta$ (shown)

	Solutions:
(ii)	$\frac{dS}{d\theta} = 18\pi\cos\theta = 0$ $\cos\theta = 0 \rightarrow \theta = 90°$ $\frac{d^2S}{d\theta^2} = -18\pi\sin\theta = -18\pi\sin(90°)$ $= -18\pi$
(iii)	Since $\frac{d^2S}{d\theta^2} < 0$, $\therefore$ the surface area will be a maximum.
11	$V = t^2 - 5t + 6 = (t-3)(t-2) = 0$
(i)	$\therefore\ t = 3\text{s}$ or $t = 2\text{s}$
(ii)	V min $\rightarrow \frac{dv}{dt} = 2t - 5 = 0$ $t = 2.5\text{s}$
(iii)	$V < 0,\ (t-3)(t-2) < 0 \rightarrow 2 < t < 3$
(iv)	$S = \frac{t^3}{3} - \frac{5t^2}{2} + 6t + c$ When $t = 0$ s = 0, $\rightarrow$ c = 0 $\therefore S = \frac{t^3}{3} - \frac{5t^2}{2} + 6t$ When $t = 2$, S = $\frac{14}{3} = 4\frac{2}{3}m$ When $t = 3$, S = $4\frac{1}{2}m$ $\therefore$ distance in the 3rd sec $= 4\frac{2}{3} - 4\frac{1}{2} = \frac{1}{6}m$
12	$B(2.5, 0)$ and $D(4, 0)$.
(i)	A1 = $\left\|\int_0^1 (x^2 - 4x)dx\right\| = \left\|\left[\frac{x^3}{3} - 2x^2\right]_0^1\right\| = \left\|-\frac{5}{3}\right\|$ A2 = $\frac{1}{2} \times 1.5 \times 3 = \frac{9}{4}$ or 2.25 A3 = $\frac{1}{2} \times 2.5 \times 5 = \frac{25}{4}$ or 6.25 A4 = $\int_4^5 (x^2 - 4x)dx = \left[\frac{x^3}{3} - 2x^2\right]_4^5 = \frac{7}{3}$ Total shaded area = A1 + A2 + A3 − A4

	Solutions:
(ii)	$= \frac{5}{3}+\frac{9}{4}+\frac{25}{4}-\frac{7}{3}=7\frac{5}{6}$ or 7.83 sq units $y = x^2 - 4x$ $\frac{dy}{dx} = 2x - 4$ $\frac{dy}{dt} = \frac{dy}{dx} \times \frac{dx}{dt}$ $-\frac{5}{6}* = (2x-4) \times \frac{5}{6}$ $2x - 4 = -1$ $2x = 3$ $x = \frac{3}{2}$