2016 AM 4E5N Prelim Paper 2 Marking Scheme

Solutions:	
1(a)	**Method 1:** $\frac{d}{dx}\ln\left(\frac{x+1}{2-x}\right)=\frac{d}{dx}\left[\ln(x+1)-\ln(2-x)\right]$ $=\frac{1}{x+1}-\frac{-1}{2-x}$ $=\frac{1}{x+1}+\frac{1}{2-x}$ **Method 2:** $\frac{d}{dx}\ln\left(\frac{x+1}{2-x}\right)=\frac{\frac{(2-x)(1)-(x+1)(-1)}{(2-x)^2}}{\frac{x+1}{2-x}}$ $=\frac{3}{(2-x)^2}\div\frac{x+1}{2-x}$ $=\frac{3}{(2-x)^2}\times\frac{2-x}{x+1}$ $=\frac{3}{(2-x)(x+1)}$
(b)(i)	**Method 1:** For $x<1$, $e^{2-x}>0$ and $-\frac{1}{2}e^{2-x}<0$. $\therefore\ \frac{dy}{dx}=-\frac{1}{2}e^{2-x}-1$ is always less than zero for $x<1$ and the curve has no stationary point. **Method 2:** For stationary point, $\frac{dy}{dx}=0$

Solutions:	
	$-\frac{1}{2}e^{2-x}-1=0$ $e^{2-x}=-2$ Since there is no solution for $e^{2-x}=-2$, thus $\frac{dy}{dx}\neq 0$. The curve has no stationary point.
(ii)	$\frac{dy}{dx}=-\frac{1}{2}e^{2-x}-1$ $y=\int\left(-\frac{1}{2}e^{2-x}-1\right)dx$ $=-\frac{1}{2}\left(\frac{e^{2-x}}{-1}\right)-x+c$ $=\frac{1}{2}e^{2-x}-x+c$ At $\left(0,\frac{1}{2}e^{2}\right)$, $\frac{1}{2}e^{2}=\frac{1}{2}e^{2-0}-0+c$ $c=0$ $\therefore$ the equation of the curve is $y=\frac{1}{2}e^{2-x}-x$.
2(i)	$\frac{d}{dx}x\cos 3x=x(-3\sin 3x)+\cos 3x$ $=-3x\sin 3x+\cos 3x$
(ii)	$\int(-3x\sin 3x+\cos 3x)\,dx=x\cos 3x+c_1$ $\int x\sin 3x\,dx=-\frac{1}{3}\left(x\cos 3x-\int\cos 3x\,dx+c_1\right)$ $=-\frac{1}{3}\left(x\cos 3x-\frac{\sin 3x}{3}+c_1+c_2\right)$

Solutions:	
	$= \frac{-x\cos 3x}{3} + \frac{\sin 3x}{9} + c$ $\int_0^{\frac{\pi}{3}} x\sin 3x \, dx = \left[\frac{-x\cos 3x}{3} + \frac{\sin 3x}{9}\right]_0^{\frac{\pi}{3}}$ $= \left[\frac{-\left(\frac{\pi}{3}\right)\cos 3\left(\frac{\pi}{3}\right)}{3} + \frac{\sin 3\left(\frac{\pi}{3}\right)}{9}\right]$ $- \left[\frac{-(0)\cos 3(0)}{3} + \frac{\sin 3(0)}{9}\right]$ $= \frac{\pi}{9}$ (shown)
3(i)	$\cos 45° = \frac{\sqrt{2} + \frac{1}{2}}{AC}$ $\frac{1}{\sqrt{2}} = \frac{\sqrt{2} + \frac{1}{2}}{AC}$ $AC = \left(2 + \frac{1}{2}\sqrt{2}\right)$ cm
(ii)	Area of the cross-section of the chocolate bar $= \frac{1}{2}\left(2 + \frac{1}{2}\sqrt{2}\right)\left(2 + \frac{1}{2}\sqrt{2}\right)$ $= \frac{1}{2}\left[2^2 + 2(2)\left(\frac{1}{2}\sqrt{2}\right) + \left(\frac{1}{2}\sqrt{2}\right)^2\right]$ $= \frac{1}{2}\left(\frac{9}{2} + 2\sqrt{2}\right)$ $= \left(\frac{9}{4} + \sqrt{2}\right)$ cm^2

Solutions:

(iii)

$$\text{Length of } AD = \frac{25+22\sqrt{2}}{\frac{9}{4}+\sqrt{2}} \times \frac{\frac{9}{4}-\sqrt{2}}{\frac{9}{4}-\sqrt{2}}$$

$$= \frac{\frac{225}{4} - 25\sqrt{2} + \frac{99}{2}\sqrt{2} - 22(2)}{\left(\frac{9}{4}\right)^2 - \left(\sqrt{2}\right)^2}$$

$$= \frac{\frac{49}{4} + \frac{49}{2}\sqrt{2}}{\frac{49}{16}}$$

$$= \left(4 + 8\sqrt{2}\right) \text{ cm}$$

4(a)

$$(2+ax)^5 = 2^5 + \binom{5}{1}2^4(ax) + \binom{5}{2}2^3(ax)^2 + \ldots$$

$$= 32 + 80ax + 80a^2x^2 + \ldots$$

$$(2+ax)^5(1+3x-2x^2)$$

$$= (32+80ax+80a^2x^2+\ldots)(1+3x-2x^2)$$

$$= 32 + 96x - 64x^2 + 80ax + 240ax^2 + 80a^2x^2 + \ldots$$

$$= 32 + (96+80a)x + (240a + 80a^2 - 64)x^2 + \ldots$$

Comparing coefficient of x term,

$$96 + 80a = -144$$

$$a = -3$$

Comparing coefficient of x^2 term,

$$240(-3) + 80(-3)^2 - 64 = b$$

$$b = -64$$

(b)

$$T_{r+1} = \binom{19}{r} 2^{19-r}(kx)^r$$

$$\text{Cofficient of } x^{11} \text{ term} = \binom{19}{11} 2^{19-11} k^{11}$$

$$= 19348992k^{11}$$

Solutions:	
	Cofficient of x^{12} term $= \binom{19}{12} 2^{19-12} k^{12}$ $= 6449664k^{12}$ $\frac{19348992k^{11}}{6449664k^{12}} = \frac{3}{5}$ $\frac{3}{k} = \frac{3}{5}$ $k = 5$
5(i)	$\frac{dy}{dx} = 3x^2 - \frac{6}{x^3}$ At $P(1, 4)$, $\frac{dy}{dx} = 3(1)^2 - \frac{6}{(1)^3}$ $= -3$ Equation of tangent is $y - 4 = -3(x - 1)$ $y = -3x + 7$ When $y = 0$, $x = \frac{7}{3}$ $\therefore$ Coordinates of A are $\left(\frac{7}{3}, 0\right)$. Gradient of normal $= \frac{1}{3}$ Equation of normal is $y - 4 = \frac{1}{3}(x - 1)$ $3y = x + 11$ When $y = 0$, $x = -11$ $\therefore$ Coordinates of B are $(-11, 0)$.
(ii)	**Method 1:** Area of triangle $APB = \frac{1}{2}(4)\left(11 + \frac{7}{3}\right)$

Solutions:	
	$= 26\frac{2}{3}$ units^2 **Method 2:** Area of triangle $APB = \frac{1}{2}\begin{vmatrix} \frac{7}{3} & 1 & -11 & \frac{7}{3} \\ 0 & 4 & 0 & 0 \end{vmatrix}$ $= 26\frac{2}{3}$ units^2
6(i)	$\frac{dy}{dx} = \frac{(2x+1)(2x) - x^2(2)}{(2x+1)^2}$ $= \frac{2x^2+2x}{(2x+1)^2}$ For stationary points, $\frac{dy}{dx} = 0$ $\frac{2x(x+1)}{(2x+1)^2} = 0$ $2x(x+1) = 0$ $x = 0$ or $x = -1$ When $x = 0$, $y = 0$ When $x = -1$, $y = -1$ $\therefore$ the coordinates of the stationary points are (0, 0) and (–1, –1).
(ii)	$\frac{d^2y}{dx^2} = \frac{(2x+1)^2(4x+2) - (2x^2+2x)[2(2x+1)(2)]}{(2x+1)^4}$ $= \frac{(2x+1)^2(4x+2) - 4(2x+1)(2x^2+2x)}{(2x+1)^4}$ $= \frac{(2x+1)[(2x+1)(4x+2) - 4(2x^2+2x)]}{(2x+1)^4}$ $= \frac{8x^2+8x+2-8x^2-8x}{(2x+1)^3}$ $= \frac{2}{(2x+1)^3}$

Solutions:	
	At (0, 0), $\frac{d^2y}{dx^2} = 2$ Since $\frac{d^2y}{dx^2} > 0$, (0, 0) is a minimum point. At (–1, –1), $\frac{d^2y}{dx^2} = -2$ Since $\frac{d^2y}{dx^2} < 0$, (–1, –1) is a maximum point.
7(i)	Centre of circle $= \left(\frac{0+1}{2}, \frac{0+2}{2}\right)$ $= \left(\frac{1}{2}, 1\right)$ Radius $= \sqrt{\left(\frac{1}{2}-1\right)^2 + (1-0)^2}$ $= \frac{\sqrt{5}}{2}$ Equation of C, $\therefore \left(x-\frac{1}{2}\right)^2 + (y-1)^2 = \frac{5}{4}$ **<u>Alternative Method</u>** Equation of circle C, $(x-a)^2 + (y-b)^2 = r^2$ ------ (1) Sub (0, 0) into equation, $a^2 + b^2 = r^2$ Sub (1, 0) into equation, $1 - 2a + a^2 + b^2 = r^2$ ------ (2) Sub (0, 2) into equation, $a^2 + 4 - 4b + b^2 = r^2$ ------ (3) Equate (1) and (2), $1 - 2a = 0 \Rightarrow a = \frac{1}{2}$

Solutions:	
	Equate (1) and (3), $4-4b=0 \Rightarrow b=1$ Sub value of *a* and of *b* into (1), $r=\frac{\sqrt{5}}{2}$ $\therefore$ Equation of circle *C*, $\left(x-\frac{1}{2}\right)^2+(y-1)^2=\frac{5}{4}$
(ii)	Substitute $y=x+k$ into equation of *C*, $\left(x-\frac{1}{2}\right)^2+(x+k-1)^2=\frac{5}{4}$ $2x^2+x(2k-3)+k^2-2k=0$ $b^2-4ac=0$ $(2k-3)^2-4(2)(k^2-2k)=0$ $4k^2-4k-9=0$ $k=\frac{-(-4)\pm\sqrt{(-4)^2-4(4)(-9)}}{2(4)}$ $k=\frac{1+\sqrt{10}}{2}$ or $k=\frac{1-\sqrt{10}}{2}$
8(i)	$\alpha+\beta=1$ $\alpha\beta=\frac{5}{2}$ $\alpha^2+\beta^2 =(\alpha+\beta)^2-2\alpha\beta$ $=-4$
(ii)	$\alpha^3+\beta^3 =(\alpha+\beta)(\alpha^2-\alpha\beta+\beta^2)$ $=-\frac{13}{2}$ or -6.5

Solutions:	
	$\alpha^3\beta^3 = (\alpha\beta)^3 = \frac{125}{8}$ or $15\frac{5}{8}$ or 15.625
(iii)	New sum of roots $= \frac{1}{\alpha^3} + \frac{1}{\beta^3}$ $= \frac{\alpha^3+\beta^3}{\alpha^3\beta^3}$ $= -\frac{52}{125}$ New product of roots $= \frac{1}{\alpha^3\beta^3} = \frac{8}{125}$ New equation: $x^2 + \frac{52}{125}x + \frac{8}{125} = 0$ **or** $125x^2 + 52x + 8 = 0$
9(i)	$\perp$ from centre bisects chord
(ii)	$\angle BAO = x$ (alt. $\angle$) $\sin\theta = \frac{OX}{7} \Rightarrow OX = 7\sin\theta$ $\cos\theta = \frac{AX}{7} \Rightarrow AX = 7\cos\theta$ $P = 7 + 14\cos\theta + 7\sin\theta + 7\cos\theta$ $= 7 + 21\cos\theta + 7\sin\theta$
(iii)	$R = \sqrt{21^2 + 7^2}$ $= 7\sqrt{10}$ $\theta = \tan^{-1}\left(\frac{7}{21}\right) = 18.435°$ or 0.322 rad $P = 7 + 7\sqrt{10}\cos(\theta - 18.4°)$ cm
(iv)	Max $P = 7 + 7\sqrt{10}$ or 29.1

Solutions:	
	$\theta - 18.435° = \cos^{-1}(1)$ $\theta = 18.4°$ or 0.322 rad
10a(i)	$-8+4a-2b+c = 8+4a+2b+c$ $b = -4$
(ii)	$1+a+b+c = 4a+c$ $a = -1$
(iii)	$27-9-12+c = 4 \Rightarrow c = -2$
10b(i)	Let $\mathrm{f}(x) = 4x^4 - 12x^3 - b^2x^2 - 7bx - 2$ $\mathrm{f}\left(-\frac{b}{2}\right) = \frac{b^4}{4} + \frac{3b^3}{2} - \frac{b^4}{4} + \frac{7b^2}{2} - 2 = 0$ $\frac{3b^3}{2} + \frac{7b^2}{2} - 2 = 0$ $3b^3 + 7b^2 - 4 = 0$ (shown)
(ii)	Let $\mathrm{g}(b) = 3b^3 + 7b^2 - 4$ Initial guess, $\mathrm{g}(-1) = 3(-1)^3 + 7(-1)^2 - 4 = 0$ By comparison, $\mathrm{g}(b) = (b+1)(3b^2 + ab - 4)$ Comparing coeff. of b^2, $3 + a = 7 \Rightarrow a = 4$ $\mathrm{g}(b) = (b+1)(3b^2 + 4b - 4)$ $= (b+1)(3b-2)(b+2)$ **Alternative method, by long division,**

Solutions:

$$
\begin{array}{r}
3b^2+4b-4 \\
b+1\,\overline{\smash{)}\,3b^3+7b^2+0b-4} \\
-\left(3b^3+3b^2\right) \qquad \\
\hline
4b^2 \qquad -4 \\
-\left(4b^2+4b\right) \quad \\
\hline
-4b-4 \\
-\left(-4b-4\right) \\
\hline
\underline{0}
\end{array}
$$

$\therefore (b+1)(3b-2)(b+2)=0$

$b=-1,\ b=\dfrac{2}{3}$ or $b=-2$

11(i) Conversion from non-linear to linear form,

$\lg y = x\lg b + \lg A$

(ii) $\lg A = 0.7$ (accept 0.65 to 0.70)

$A = 10^{0.7} = 5.01$ (3 s.f.) (accept 4.47 to 5.01)

Gradient $= \lg b$

$= \dfrac{2.16-0.991}{10-8}$

$b = 10^{0.146} = 1.40$ (3 s.f.)

(iii) $A=\left(\dfrac{2}{b}\right)^x$ → $Ab^x = 2^x$

Plot $\lg y = x\lg 2$

Plotting of correct straight line

From the graph,

$x = 4.6$ (accept 4.2 to 4.6)

Solutions: