# ANDERSON SECONDARY SCHOOL
# Preliminary Examination 2017
# Secondary Four Express and Five Normal Academic

## ADDITIONAL MATHEMATICS PAPER 1
## ANSWERS

**1** **(i)** $3y = 15x - 2(x-3)^{\frac{3}{2}} - 44$ or $y = 5x - \frac{2}{3}(x-3)^{\frac{3}{2}} - \frac{44}{3}$

**(ii)** $3y = 52 - x$

**3** **(i)** $p = 4$ , $q = -1$

**(ii)** $A = (0, 3)$, $B = \left(\frac{3}{2}, 0\right)$, $C = \left(\frac{5}{2}, 0\right)$

**4** **(i)** $52 - 30\sqrt{3}$ **(ii)** $30\sqrt{3} - 51$

**5** Case 1: right-angle at *B*

**(i)** $(3, 1)$ **(ii)** $12.5$ units$^2$

<u>Case 2: right-angle at *P*</u>

**(i)** $(2\frac{4}{5}, \frac{3}{5})$ **(ii)** $13$ units$^2$

**6** **(ii)** $y = 51.6°$ (1dp)

**7** **(i)** $a = 5$, $b = 0.5$, $c = -4$

**(ii)** $P = (-8.14, 0)$ , $Q = (1.85, 0)$ and $R = (4.32, 0)$

**8** **(ii)** $(1, 0)$ and $\left(-\frac{1}{3}, 0\right)$ **(iv)** $-4 < m < 0$

**9** **(ii)** $\int x \sin 6x \, dx = \frac{x}{6} + \frac{\sin 6x}{36} - \frac{x \cos^2 3x}{3} + c$

**10** **(ii)** $\left(\frac{4}{9}, \frac{1}{3}\right)$

**11** **(i)** $2 + \frac{3}{2x-1} + \frac{2}{x+5} - \frac{1}{(x+5)^2}$

**12** **(ii)** $\frac{\sqrt{1600 + (60-x)^2}}{2} + \frac{x}{4}$ seconds

**(iii)** $\frac{dT}{dx} = \frac{x - 60}{2\sqrt{1600 + (60-x)^2}} + \frac{1}{4}$

**(iv)** $x = 36.9$

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## ADDITIONAL MATHEMATICS PAPER 1
## MARKING SCHEME

**1** A curve is such that $\frac{\mathrm{d}y}{\mathrm{d}x} = 5 - \sqrt{x-3}$ and $A$ (7, 15) is a point on the curve.

**(i)** Find the equation of the curve. [2]

**(ii)** Find the equation of the normal to the curve at $A$. [2]

Marking Scheme

**(i)** $\frac{\mathrm{d}y}{\mathrm{d}x} = 5 - \sqrt{x-3}$

$y = 5x - \frac{2}{3}(x-3)^{\frac{3}{2}} + c$ **[M1]**

When $x = 7$, $y = 15$:

$15 = 5(7) - \frac{2}{3}(7-3)^{\frac{3}{2}} + c$

$15 = 35 - \frac{16}{3} + c$

$c = -\frac{44}{3}$

$3y = 15x - 2(x-3)^{\frac{3}{2}} - 44$ or $y = 5x - \frac{2}{3}(x-3)^{\frac{3}{2}} - \frac{44}{3}$ **[A1]**

**(ii)** $\frac{\mathrm{d}y}{\mathrm{d}x} = 5 - \sqrt{x-3}$

When $x = 7$,

$\frac{\mathrm{d}y}{\mathrm{d}x} = 5 - \sqrt{7-3} = 3$

Gradient of normal $= -\frac{1}{3}$ **[M1]**

Equation of normal : $y - 15 = -\frac{1}{3}(x-7)$ **[A1]**

or $3y = 52 - x$

**2** The equation of a curve is given by $y = x^2 + 2ax + 2a - 3$, where $a$ is a constant. Show that, for all values of $a$, the curve intersects the $x$-axis at two distinct points. [4]

Marking Scheme

At the points where the curve intersects the $x$-axis,

$$x^2 + 2ax + 2a - 3 = 0$$

Discriminant $= (2a)^2 - 4(1)(2a-3)$ **[M1]**

$= 4a^2 - 8a + 12$

$= 4(a^2 - 2a + 3)$

$= 4[(a-1)^2 + 2]$

$= 4(a-1)^2 + 8$ **[M1]**

Since $(a-1)^2 \geq 0$ for all real $a$,

$4(a-1)^2 \geq 0$ **[M1]**

$4(a-1)^2 + 8 > 0$ for all $a$.

Since discriminant > 0 for all $a$,

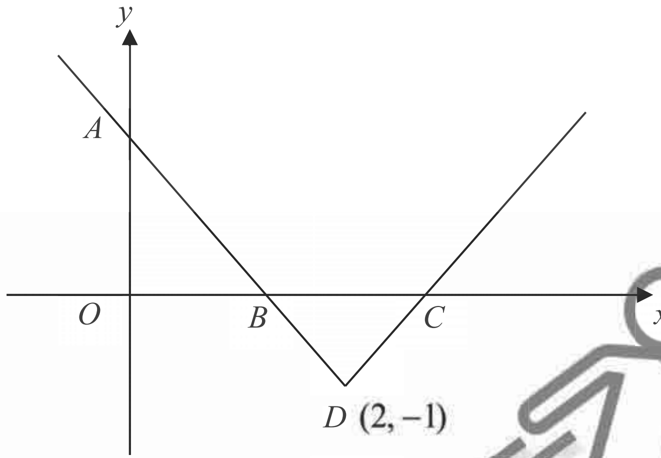
the eqn $x^2 + 2ax + 2a - 3 = 0$ has 2 distinct roots for all values of $a$. **[A1]**

Thus the curve intersects the $x$ axis at two distinct points.

**3** The diagram shows the graph of $y = | p - 2x | + q$, where $p$ and $q$ are integers. $A$ is the point where the graph intersects the $y$-axis, and $B$ and $C$ are the points where it intersects the $x$-axis. Point $D(2, -1)$ is the vertex of the graph.

**(i)** Find the value of $p$ and of $q$. [2]

**(ii)** Hence find the coordinates of $A$, $B$ and $C$. [4]




Marking Scheme

**(i)** $p = 4$ **[B1]**

$q = -1$ **[B1]**

**(ii)** $y = | 4 - 2x | - 1$

When $x = 0$, $y = | 4 | - 1 = 3$

Thus $A = (0, 3)$ **[B1]**

When $y = 0$,

$| 4 - 2x | - 1 = 0$

$| 4 - 2x | = 1$

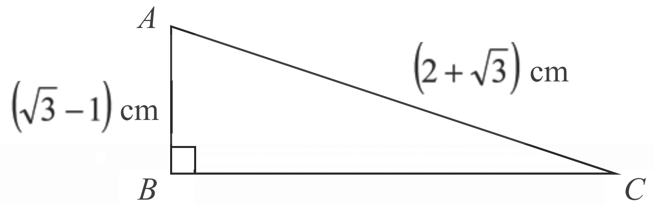
$4 - 2x = 1$ or $4 - 2x = -1$ **[M1]**

$x = \frac{3}{2}$ or $x = \frac{5}{2}$ **[M1]**

Therefore $B = \left(\frac{3}{2}, 0\right)$ and $C = \left(\frac{5}{2}, 0\right)$ **[A1]**

**4** **(i)** Express $\left(\dfrac{1-\sqrt{3}}{2+\sqrt{3}}\right)^2$ in the form $a+b\sqrt{3}$ where $a$ and $b$ are integers. [3]

**(ii)** The diagram shows a triangle $ABC$ where angle $ABC = 90°$, $AB = \left(\sqrt{3}-1\right)$cm and $AC = \left(2+\sqrt{3}\right)$cm . Using your answer to **(i)**, find the exact value of $\cos^2(A\hat{C}B)$ without using a calculator. [3]




Marking Scheme

**(i)** $\left(\dfrac{1-\sqrt{3}}{2+\sqrt{3}}\right)^2 = \left(\dfrac{1-\sqrt{3}}{2+\sqrt{3}} \times \dfrac{2-\sqrt{3}}{2-\sqrt{3}}\right)^2$ **[M1]** *rationalize denominator*

$= \left(5-3\sqrt{3}\right)^2$ **[M1]** *correct multiplication*

$= 52-30\sqrt{3}$ **[A1]** *correct evaluation of square*

**(ii)** $\cos^2(A\hat{C}B) = 1-\sin^2(A\hat{C}B)$

$= 1-\left(\dfrac{\sqrt{3}-1}{2+\sqrt{3}}\right)^2$ **[M1]** *Use of trigo identity*

$= 1-\left(\dfrac{1-\sqrt{3}}{2+\sqrt{3}}\right)^2$

$= 1-\left(52-30\sqrt{3}\right)$ **[M1]** *Use of answer in* ***(i)*** *- allow ecf*

$= 30\sqrt{3}-51$ **[A1]**

**5** It is given that point $A\,(8,11)$ lies on the line $l$ with equation $y = 2x - 5$, and $P$ is the point (1, 2).

If $B$ is the point on $l$ such that $PBA$ is a right-angled triangle, find

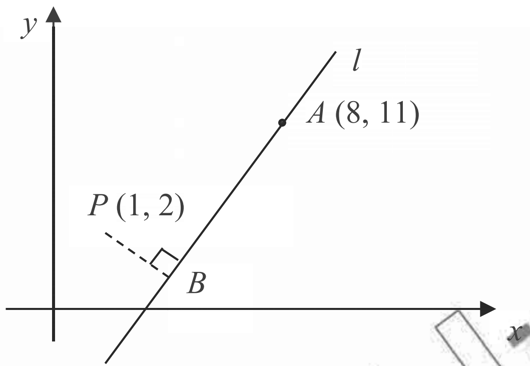
**(i)** the coordinates of $B$, [5]

**(ii)** the area of triangle $PBA$. [2]

Marking Scheme

**Case 1: right-angle at *B***

**(ii)**




Since $PB$ is perpendicular to line $l$, gradient of $PB = -\frac{1}{2}$ **[M1]**

Since $B$ is on Line $l$, its coordinates satisfy the equation.
Let coordinates of $B$ be $(x, 2x-5)$. **[M1]**

Gradient of $PB = -\frac{1}{2} \Rightarrow \frac{(2x-5)-2}{x-1} = -\frac{1}{2}$ **[M1]**

$\Rightarrow 2x - 7 = -\frac{1}{2}x + \frac{1}{2}$

$\Rightarrow x = 3$, **[M1]** *solving for one of the coordinates correctly*
$y = 1$

Thus the coordinates of $B = (3, 1)$ **[A1]**

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***[Alternative solution]***

Since $PB$ is perpendicular to Line $l$, gradient of $PB = -\frac{1}{2}$ **[M1]**

Equation of $PB$ : $y = -\frac{1}{2}x + c$

Sub in (1, 2), equation of $PB$ :

$y = -\frac{1}{2}x + \frac{5}{2}$ --- (1) **[M1]** *finding eqn of PB correctly*

$y = 2x - 5$ --- (2)

Sub (1) into (2),

$-\frac{1}{2}x + \frac{5}{2} = 2x - 5$ **[M1]** *or equivalent*

$x = 3$ **[M1]** *solving for one of the coordinates correctly*

$y = 1$

Thus the coordinates of $B = (3, 1)$ **[A1]**

**(ii)** Considering area of trapeziums formed by the 3 points and the $x$-axis,

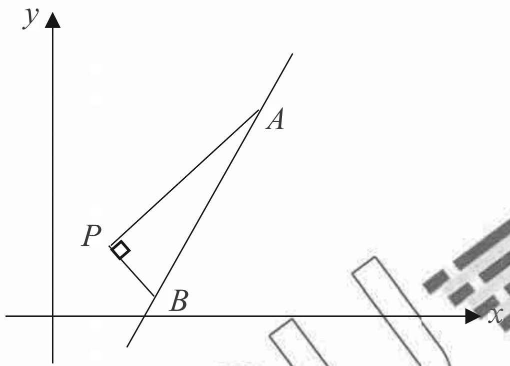
Area of $PBA = \frac{1}{2}(2+11)(7) - \frac{1}{2}(2+1)(2) - \frac{1}{2}(1+11)(5)$ **[M1]**

$= 12.5$ units$^2$ **[A1]**

***(Also accept Shoelace Mtd or Area of triangle method)***

**<u>Case 2: right-angle at *P*</u>**

**(ii)**




Gradient of $AP = \frac{9}{7}$

Gradient of $PB = -\frac{7}{9}$ **[M1]**

Equation of $PB$ : $y = -\frac{7}{9}x + c$

Sub in (1, 2), equation of $PB$ :

$y = -\frac{7}{9}x + \frac{25}{9}$ --- (1) **[M1]** *finding eqn of PB correctly*

$y = 2x - 5$ --- (2)

Sub (1) into (2),

$-\frac{7}{9}x + \frac{25}{9} = 2x - 5$ **[M1]** *or equivalent*

$x = 2\frac{4}{5}$ , **[M1]** *solving for one of the coordinates correctly*

$y = \frac{3}{5}$

Thus the coordinates of $B = (2\frac{4}{5}, \frac{3}{5})$ **[A1]**

**(ii)** Area of $PBA = \frac{1}{2}\left(\sqrt{130}\right)\left(\frac{\sqrt{130}}{5}\right)$ **[M1]**

$= 13$ units$^2$ **[A1]**

**6** **(i)** Prove that $(\sin 2y+2)(\sin y-\cos y)=2\cos^3 y\,(\tan^3 y-1)$. [4]

**(ii)** Hence find the acute angle $y$, in degrees, such that
$(\sin 2y+2)(\sin y-\cos y)=2\cos^3 y$. [2]

Marking Scheme

**(i)** $(\sin 2y+2)(\sin y-\cos y)$

$=(2\sin y\cos y+2)(\sin y-\cos y)$ **[M1]**

$=2(\sin^2 y\cos y-\sin y\cos^2 y+\sin y-\cos y)$

$=2\left[\sin y(1-\cos^2 y)-\cos y(1-\sin^2 y)\right]$ **[M1]**

$=2\left[\sin y(\sin^2 y)-\cos y(\cos^2 y)\right]$ **[M1]**

$=2\left[\sin^3 y-\cos^3 y\right]$

$=2\cos^3 y\left(\dfrac{\sin^3 y}{\cos^3 y}-\dfrac{\cos^3 y}{\cos^3 y}\right)$

$=2\cos^3 y\,(\tan^3 y-1)$ (Proven) **[A1]**

***Alternative solution:***

RHS = $2\cos^3 y\,(\tan^3 y-1)$

$=2\cos^3 y\left(\dfrac{\sin^3 y}{\cos^3 y}-1\right)$ **[M1]**

$=2\sin^3 y-2\cos^3 y$

$=2\left[(1-\cos^2 y)\sin y-(1-\sin^2 y)\cos y\right]$ **[M1]**

$=2\left(\sin y-\sin y\cos^2 y-\cos y+\sin^2 y\cos y\right)$

$=2\left[\sin y\cos y(\sin y-\cos y)+\sin y-\cos y\right]$

$=2\left[(\sin y\cos y+1)(\sin y-\cos y)\right]$ **[M1]**

$=(2\sin y\cos y+2)(\sin y-\cos y)$

$=(\sin 2y+2)(\sin y-\cos y)$ (Proven) **[A1]**

**(ii)** $(\sin 2y+2)(\sin y-\cos y)=2\cos^3 y$

$2\cos^3 y\,(\tan^3 y-1)=2\cos^3 y$

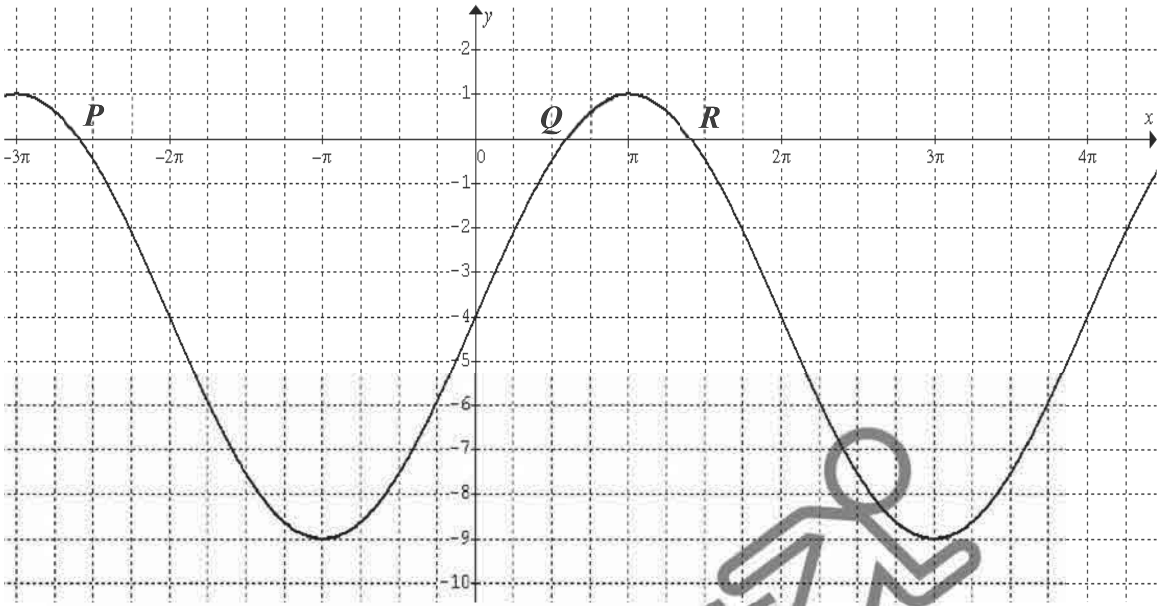
Since $y$ is acute, $\cos y>0$

$\Rightarrow\ \tan^3 y-1=1$ **[M1]**

$\Rightarrow\ \tan^3 y=2$

$\Rightarrow\ y=51.6^\circ$ (1dp) **[A1]**

**7** The figure shows part of the graph of $y = a\sin(bx) + c$. Points $P$, $Q$ and $R$ on the graph lie on the $x$ axis.




**(i)** Find the value of each of the constants $a$, $b$ and $c$. [3]

**(ii)** Hence find the coordinates of $P$, $Q$ and $R$. [4]

<u>Marking Scheme</u>

**(i)** $a = 5$ **[B1]**

$b = 0.5$ **[B1]**

$c = -4$ **[B1]**

**(ii)** $y = 5\sin\frac{x}{2} - 4$

When $y = 0$,

$$5\sin\frac{x}{2} - 4 = 0$$

$$\sin\frac{x}{2} = \frac{4}{5}$$

$$\frac{x}{2} = -\pi - 0.9272,\quad 0.9272,\quad \pi - 0.9272,\ \ldots$$ **[M1]**

$$x = -8.14,\quad 1.85,\quad 4.43,\ \ldots$$

Therefore $P = (-8.14, 0)$ , $Q = (1.85, 0)$ and $R = (4.32, 0)$ **[A3]**

**8** A curve has the equation $y = 4-(3x-1)^2$.

**(i)** Explain why the highest point on the curve has coordinates $\left(\frac{1}{3}, 4\right)$. [1]

**(ii)** Find the coordinates of the points at which the curve intersects the $x$-axis. [2]

**(iii)** Sketch the graph of $y = \left|4-(3x-1)^2\right|$. [2]

**(iv)** The equation $\left|4-(3x-1)^2\right| = mx+4$ has 4 distinct solutions. Using your graph, determine the range of values of $m$. [2]

<u>Marking Scheme</u>

**(i)** Since $(3x-1)^2 \geq 0$, $-(3x-1)^2 \leq 0$

$$\Rightarrow \quad 4-(3x-1)^2 \leq 4.$$

Thus the maximum value of $y$ is 4, and it occurs when $x = \frac{1}{3}$.

**[B1]** *(or other equiv logical arguments )*

Therefore the highest point on the curve has coordinates $\left(\frac{1}{3}, 4\right)$.

**(ii)** When $y = 0$,

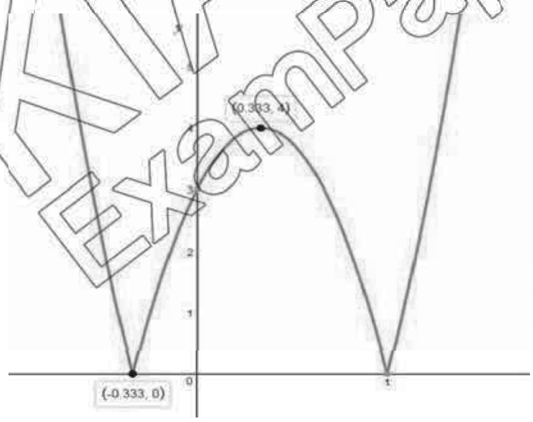
$4-(3x-1)^2 = 0$

$3x-1 = 2$ or $3x-1 = -2$ **[M1]**

$x = 1$ or $x = -\frac{1}{3}$

The curve intersects the $x$-axis at the points $(1, 0)$ and $\left(-\frac{1}{3}, 0\right)$. **[A1]**

**(iii)**




**[C1]** Shape of Graph

**[C1]** Passes through (0, 3), $\left(-\frac{1}{3}, 0\right)$, (1, 0) and $\left(\frac{1}{3}, 4\right)$.

**(iv)** Consider intersection of curve in (iii) with the line $y = mx+4$.
When $m = 0$, there're 3 points of intersection. For 4 intersection points, $m < 0$.

Gradient of line that passes through (0, 4) and (1, 0) $= -4$.
Thus range of values of $m$ is $-4 < m < 0$ **[B2]**

**9** **(i)** Show that $\frac{d}{dx}\left(x\cos^2 3x\right) = \cos^2 3x - 3x\sin 6x$. [3]

**(ii)** Hence integrate $x\sin 6x$ with respect to $x$. [4]

Marking Scheme

**(i)** $\frac{d}{dx}\left(x\cos^2 3x\right) = \cos^2 3x + x(2\cos 3x)(-\sin 3x)(3)$ **[M1]** *product rule +*

**[M1]** *chain rule*

$= \cos^2 3x - 3x(2\cos 3x \sin 3x)$

$= \cos^2 3x - 3x\sin 6x$ **[A1]** *double angle formula*

**(ii)** From **(i),**

$\int (\cos^2 3x - 3x\sin 6x)\, dx = x\cos^2 3x + c_1$ **[M1]** *reverse of differentiation*

$\int \cos^2 3x\, dx - \int 3x\sin 6x\, dx = x\cos^2 3x + c_1$

$\int \frac{1+\cos 6x}{2}\, dx - \int 3x\sin 6x\, dx = x\cos^2 3x + c_1$ **[M1]** *double angle formula*

$\frac{1}{2}x + \frac{\sin 6x}{(2)(6)} - \int 3x\sin 6x\, dx = x\cos^2 3x + c$ **[M1]** *correct integration*

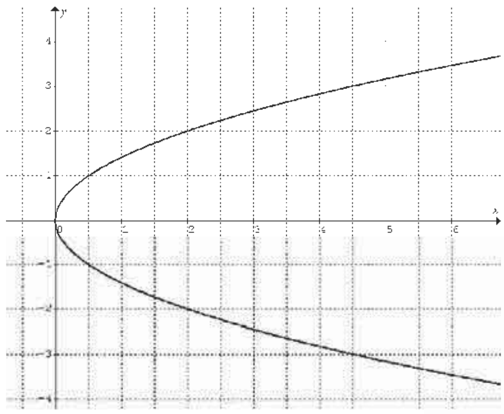
$\int x\sin 6x\, dx = \frac{x}{6} + \frac{\sin 6x}{36} - \frac{x\cos^2 3x}{3} + c$ **[A1]**

**10** **(i)** Sketch the parabola $y^2 = 2x$. [2]

**(ii)** The curve $y^2 = 2x$ intersects the straight line $y = 3x - 1$ at the points $A$ and $B$. Find the coordinates of the midpoint of $AB$. [6]

Marking Scheme

**(i)**



**[C1]** Correct shape of curve

**[C1]** passes through origin

**(ii)** $y = 3x - 1 \quad -(1)$

$y^2 = 2x \quad -(2)$

From (1), $x = \dfrac{y+1}{3}$

Sub into (2):

$y^2 = 2\left(\dfrac{y+1}{3}\right)$ **[M1]**

$3y^2 - 2y - 2 = 0$ **[M1]**

$y = \dfrac{2 \pm \sqrt{4 - 4(3)(-2)}}{6}$

$y = \dfrac{1 \pm \sqrt{7}}{3}$ **[M1]**

When $y = \dfrac{1+\sqrt{7}}{3}$, $x = \dfrac{\frac{1+\sqrt{7}}{3} + 1}{3} = \dfrac{4+\sqrt{7}}{9}$.

When $y = \dfrac{1-\sqrt{7}}{3}$, $x = \dfrac{\frac{1-\sqrt{7}}{3} + 1}{3} = \dfrac{4-\sqrt{7}}{9}$.

Coordinates of $A$ and $B$ are $\left(\dfrac{4+\sqrt{7}}{9}, \dfrac{1+\sqrt{7}}{3}\right)$ and $\left(\dfrac{4-\sqrt{7}}{9}, \dfrac{1-\sqrt{7}}{3}\right)$. **[M1]**

Midpoint of $AB = \left(\dfrac{\frac{4+\sqrt{7}}{9} + \frac{4-\sqrt{7}}{9}}{2}, \dfrac{\frac{1+\sqrt{7}}{3} + \frac{1-\sqrt{7}}{3}}{2}\right)$ **[M1]**

$= \left(\dfrac{4}{9}, \dfrac{1}{3}\right)$ **[A1]** *(Only exact answer accepted)*

**11** **(i)** Express $\dfrac{4x^3+45x^2+126x+16}{(2x-1)(x+5)^2}$ in partial fractions. [5]

**(ii)** Hence show that

$$\int_1^2 \frac{4x^3+45x^2+126x+16}{(2x-1)(x+5)^2}\,\mathrm{d}x = \frac{83}{42}+\frac{\ln 27}{2}+\ln\left(\frac{49}{36}\right).$$ [4]

Marking Scheme

**(i)** By long division,

$$\frac{4x^3+45x^2+126x+16}{(2x-1)(x+5)^2}$$

$$= 2+\frac{7x^2+46x+66}{(2x-1)(x+5)^2}$$ **[B1]** quotient of 2 + **[M1]** proper fraction

$$= 2+\frac{A}{2x-1}+\frac{B}{x+5}+\frac{C}{(x+5)^2}$$ **[M1]** (correct general form of partial frac)

$$= 2+\frac{3}{2x-1}+\frac{2}{x+5}-\frac{1}{(x+5)^2}$$ **[A2]** (–1 if 1-2 mistakes in $A$, $B$, $C$ values)

***[Alternative Solution]***

By inspection / observation,

$$\frac{4x^3+45x^2+126x+16}{(2x-1)(x+5)^2} = 2+\frac{A}{2x-1}+\frac{B}{x+5}+\frac{C}{(x+5)^2}$$

**[B1]** quotient of 2 + **[M1]** correct general form of partial frac

Multiplying throughout by $(2x-1)(x+5)^2$:

$4x^3+45x^2+126x+16$

$= 2(2x-1)(x+5)^2 + A(x+5)^2 + B(2x-1)(x+5) + C(2x-1)$ **[M1]**

By substituting suitable values of $x$, $A=3$, $B=2$ and $C=-1$.

Therefore $\dfrac{4x^3+45x^2+126x+16}{(2x-1)(x+5)^2} = 2+\dfrac{3}{2x-1}+\dfrac{2}{x+5}-\dfrac{1}{(x+5)^2}$

**[A2]** (–1 if 1-2 mistakes in $A$, $B$, $C$ values)

**(ii)** $\displaystyle\int_1^2 \frac{4x^3+45x^2+126x+16}{(2x-1)(x+5)^2}\,dx$

$$=\int_1^2 \left(2+\frac{3}{2x-1}+\frac{2}{x+5}-\frac{1}{(x+5)^2}\right)dx$$

$=\left[2x+\frac{3}{2}\ln(2x-1)+2\ln(x+5)+\frac{1}{x+5}\right]_1^2$ **[M1]** *for ln terms* **+ [M1]** *for* $\frac{1}{x+5}$

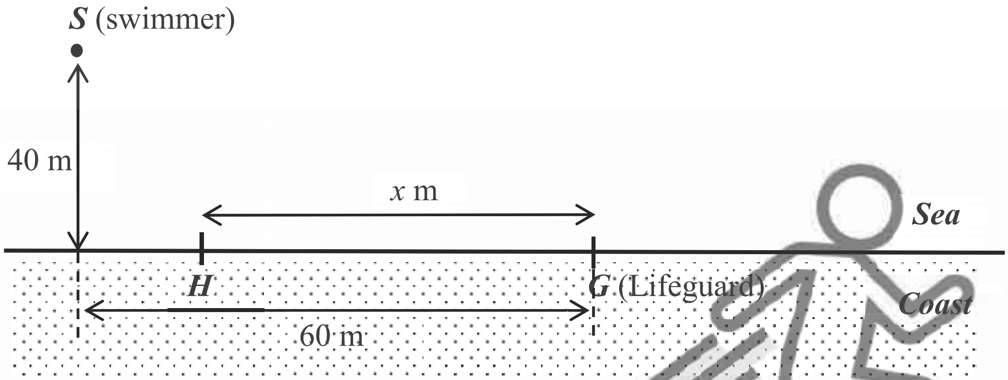
$=\left[4+\frac{3}{2}\ln(3)+2\ln(7)+\frac{1}{7}\right]-\left[2+\frac{3}{2}\ln(1)+2\ln(6)+\frac{1}{6}\right]$ **[M1]** *subn of limits*

$$=\frac{83}{42}+\frac{3}{2}\ln(3)+\ln 49-\ln 36$$

$=\frac{83}{42}+\frac{\ln 27}{2}+\ln\left(\frac{49}{36}\right)$ (Shown) **[A1]** *for correct appln of laws of log*

**12** A lifeguard at a beach resort is stationed at point *G* along the coastline, as shown in the diagram below. When he detects a swimmer who needs help at a point *S*, he would run along the coastline over a distance of $x$ m to a point *H,* and then swim in a straight line, *HS*, towards the swimmer. The lifeguard runs at a speed of 4 m/s and swims at a speed of 2 m/s.

A swimmer in distress is detected at a position that is 40 m away from the coastline, and the foot of perpendicular from the swimmer to the coastline is at a distance of 60 m away from the lifeguard.




**(i)** Show that the time taken by the lifeguard to swim from *H* to *S* is $\dfrac{\sqrt{1600+(60-x)^2}}{2}$ seconds. [2]

**(ii)** Find, in terms of $x$, the total time $T$ taken by the lifeguard to reach the swimmer. [1]

**(iii)** Obtain an expression for $\dfrac{\mathrm{d}T}{\mathrm{d}x}$. [2]

**(iv)** Find the value of $x$ such that the lifeguard would be able to reach the swimmer in the shortest possible time. [4]

<u>Marking Scheme</u>

**(i)** Using Pythagora's Thm,

$$HS = \sqrt{40^2 + (60-x)^2}$$

$$= \sqrt{1600+(60-x)^2} \text{ m} \qquad \textbf{[M1]}$$

Time taken by the lifeguard to swim from *H* to *S*

$$= \frac{\text{Distance Travelled}}{\text{Speed}} = \frac{\sqrt{1600+(60-x)^2}}{2} \text{ s.} \quad \textbf{[A1]}$$

**(ii)** Time taken to travel from *G* to *H* = $\dfrac{\text{Distance Travelled}}{\text{Speed}} = \dfrac{x}{4}$ seconds

Total time, $T = \dfrac{\sqrt{1600+(60-x)^2}}{2} + \dfrac{x}{4}$ seconds **[B1]**

**(iii)** $\dfrac{dT}{dx}=\dfrac{2(60-x)(-1)}{4\sqrt{1600+(60-x)^2}}+\dfrac{1}{4}$

$\dfrac{dT}{dx}=\dfrac{x-60}{2\sqrt{1600+(60-x)^2}}+\dfrac{1}{4}$ **[B2]**

**(iv)** When $T$ is stationary, $\dfrac{dT}{dx}=0$ .

$\dfrac{x-60}{2\sqrt{1600+(60-x)^2}}+\dfrac{1}{4}=0$ **[M1]**

$\dfrac{x-60}{2\sqrt{1600+(60-x)^2}}=-\dfrac{1}{4}$

$-2x+120=\sqrt{1600+(60-x)^2}$

$(2x-120)^2=1600+(60-x)^2$

$3x^2-360x+9200=0$ **[M1]**

$x=\dfrac{360\pm\sqrt{360^2-4(3)(9200)}}{6}$

$x=\dfrac{360\pm\sqrt{19200}}{6}$

$x=36.9$ or 83.1 (reject since $x$ cannot be larger than 60.) **[M1]**

First derivative test:

| $x$ | $36.9^-$ | 36.9 | $36.9^+$ |
|---|---|---|---|
| $\dfrac{dT}{dx}$ | Negative | 0 | Positive |
| Slope | \ | – | / |

Thus $x=36.9$ gives the min time taken to reach the swimmer. **[A1]**

***End of Marking Scheme***

**Secondary 4E5N**
**Preliminary Examination 2017**
**ADDITIONAL MATHEMATICS**
**Answer Keys for Paper 2**

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**1** **(i)** $p = -\frac{2}{3}$ or 2

**(ii)** 8

**2** **(i)** Proof

**(ii)** $x = \frac{\pi}{6}, \frac{5\pi}{6}, \pi, \frac{7\pi}{6}$ or $\frac{11\pi}{6}$

**3** **(i)** $\alpha\beta = 2$

**(ii)** $\alpha + \beta = -4$

**(iii)** $\alpha - \beta = -\sqrt{8}$
$(x+4)(x+2\sqrt{2}) = 0$

**4** **(i)** Use corresponding $\angle$s, $PD$ // $BC$ **AND** $\angle$s in alternate segment.

**(ii)** Use the result of (i).

**(iii)** Use $\angle$s in the same segment, alternate $\angle$s, $PD$ // $BC$ **AND** $\angle$s in alternate segment.

**5** **(i)** $-\frac{6x+1}{e^{3x}}$

**(ii)** $x > -\frac{1}{6}$

**(iii)** $-\frac{e^3}{28}$ units/s

**6** **(i)** $(2x+1)(x+2)(x-2)$

**(ii)** $(2x+1)(x^2+1) = 0$
Since $x^2 + 1 > 0,\ 2x + 1 = 0$.
$\therefore$ The equation has only one solution i.e. $x = -\frac{1}{2}$.

**(iii)** $k < 8\frac{1}{6}$

**7** **(i)** $x$-coordinates of $A$ and $B$ are 2 and 1 respectively.

**(ii)** $\frac{1}{2}$ units$^2$

**(iii)** The curve $x = 3y^2 - 8y + 6$ is a **reflection (or mirror image)** of the curve $y = 3x^2 - 8x + 6$ in the line $y = x$. $\therefore$ the area bounded by the curve $x = 3y^2 - 8y + 6$ and the line $y = x$ is also $\frac{1}{2}$ units$^2$.

**8** **(i)** $-8\sqrt{3}$ cm/s

**(ii)** $t = \frac{\pi}{2}$,

**(iii)** 18.3 cm

**9** **(i)** $y = -\frac{1}{5}x + 3$

**(ii)** Equation of $AG$ is $y = -\frac{3}{2}x - \frac{7}{2}$

Coordinates of $G = (-5, 4)$

**(iii)** $x^2 + y^2 + 10x - 8y + 28 = 0$

**(iv)** Coordinates of $H = (-1, -2)$.

**(v)** Equation of circle $C_2$ is $(x+1)^2 + (y+2)^2 = 13$

**10** **(a)** **(i)** Show that $\sqrt{3 - e^x} + 1 - ke^x \neq 0$ or $\sqrt{3 - e^x} + 1 \neq ke^x$

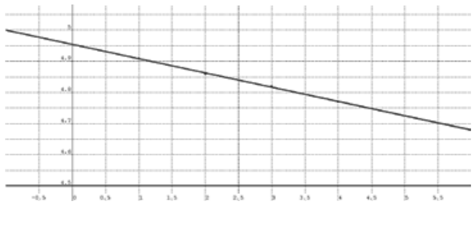
**(ii)** $\ln 3$ or $\ln 2$

**(b)** **(i)** $a = 4, b = \ln 2$

**(ii)** $x = \frac{2^t}{4 + 2^t}$

**11** **(i)** $\lg V = (\lg a)t + \lg V_0$

| $t$ | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| $\lg V$ | 4.91 | 4.86 | 4.82 | 4.77 |



**(ii)** $a = 0.900$ (3sf), $V_0 = 90100$(3sf). Mr Lee paid \$90100 for the car.

**(iii)** \$53200

**Secondary 4E5N**
**Preliminary Examination 2017**
**ADDITIONAL MATHEMATICS**
**Paper 2 Marking Scheme**

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**1** **(i)** $(1-2x)^2(1+px)^7 = (1-4x+4x^2)(1+7px+21p^2x^2+.........)$ **[M1]**

Coefficient of $x^2 = 32$

$21p^2 - 28p + 4 = 32$ **[M1]**

$21p^2 - 28p - 28 = 0$

$3p^2 - 4p - 4 = 0$ **[M1]**

$(3p+2)(p-2) = 0$

$\underline{\underline{p = -\frac{2}{3} \text{ or } 2}}$ **[A1]**

**(ii)** Since coefficient of $x^2$ in $(1-2x)^2(1+px)$ is 32,

Coefficient of $x^2$ in $(1-x)^2\left(1+\frac{px}{2}\right)^7 = \left(\frac{1}{2}\right)^2 \times 32$ **[M1]**

$= \underline{\underline{8}}$ **[A1]**

**2** **(i)** $\sin 3x \equiv \sin(2x+x)$,

$\equiv \sin 2x \cos x + \cos 2x \sin x$ **[M1]**

$\equiv 2\sin x \cos^2 x + (1-2\sin^2 x)\sin x$

$\equiv (2\sin x)(1-\sin^2 x) + \sin x - 2\sin^3 x$ **[M1]**

$\equiv 3\sin x - 4\sin^3 x$ **[A1]**

**(ii)** $\sin 3x = 2\sin x,\ 0 < x < 2\pi$

$3\sin x - 4\sin^3 x = 2\sin x$

$4\sin^3 x - \sin x = 0$ **[M1]**

$\sin x(4\sin^2 x - 1) = 0$

$\sin x = 0 \text{ or } \pm\frac{1}{2}$ **[M1]**

When $\sin x = 0,\ x = \pi$. **[A1]**

When $\sin x = \frac{1}{2},\ x = \frac{\pi}{6} \text{ or } \frac{5\pi}{6}$. **[A1]**

When $\sin x = -\frac{1}{2},\ x = \frac{7\pi}{6} \text{ or } \frac{11\pi}{6}$. **[A1]**

$\underline{\underline{x = \frac{\pi}{6}, \frac{5\pi}{6}, \pi, \frac{7\pi}{6} \text{ or } \frac{11\pi}{6}}}$ (accept $x = 0.524,\ 2.62, 3.14, 3.67 \text{ or } 5.76$)

3 $x^2 = 12x - 4$

$x^2 - 12x + 4 = 0$

(i) $\alpha^2\beta^2 = 4$ **[M1]**

$\alpha\beta = \pm 2$

Since $\alpha < 0$ & $\beta < 0, \alpha\beta > 0$.

$\therefore \alpha\beta = 2$ **[A1]**

(ii) $\alpha^2 + \beta^2 = 6$ **[M1]**

$(\alpha+\beta)^2 - 2\alpha\beta = 12$

$(\alpha+\beta)^2 = 12 + 2(2)$

$= 16$ **[M1]**

$\alpha + \beta = \pm 4$

Since $\alpha < 0, \beta < 0, \alpha + \beta < 0$,

$\therefore \alpha + \beta = -4$ **[A1]**

(iii) $(\alpha - \beta)^2 = \alpha^2 + \beta^2 - 2\alpha\beta$

$= 12 - 2(2)$

$= 8$ **[M1]**

$\alpha - \beta = \pm\sqrt{8}$

Since $\alpha < \beta$, $\alpha - \beta < 0$.

$\therefore \alpha - \beta = -\sqrt{8}$ **[M1]**

Quadratic equation with roots $\alpha+\beta$ and $\alpha-\beta$ is $(x+4)(x+2\sqrt{2}) = 0$ **[A1]**

4 (i) $\angle ADP = \angle ACB$ (corresponding $\angle$ s, $PD \, // \, BC$) **[M1]**

$= \angle ABP$ ($\angle$ s in alternate segment) **[M1]**

(ii) Since $\angle ADP = \angle ABP$ from (i), using angles in the segment, *A*, *D*, *B* and *P* lie on a circle. **[M1]**

(iii) $\angle BAP = \angle BDP$ ($\angle$ s in the same segment) **[M1]**

$= \angle DBC$ (alternate $\angle$ s, $PD \, // \, BC$) **[M1]**

$\angle BAP = \angle BCD$ ($\angle$ s in alternate segment) **[M1]**

Since $\angle DBC = \angle BCD$,

$\therefore DB = DC$ **[A1]**

**5** $y = \frac{1+2x}{e^{3x}}$

**(i)** $\frac{dy}{dx} = \frac{e^{3x}(2)-(1+2x)(3e^{3x})}{e^{6x}}$ **[M1]**

$= \frac{2-3-6x}{e^{3x}}$

$= -\frac{6x+1}{e^{3x}}$ **[A1]** (accept $\frac{-6x-1}{e^{3x}}$)

**(ii)** For $y$ to be decreasing, $\frac{dy}{dx} < 0$.

$-\frac{6x+1}{e^{3x}} < 0$ **[M1]**

Since $e^{3x} > 0$, **[M1]**

$-(6x+1) < 0$.

$x > -\frac{1}{6}$ **[A1]**

**(iii)** $\frac{dy}{dt} = \frac{dy}{dx} \times \frac{dx}{dt}$

$= -\frac{6x+1}{e^{3x}} \times \frac{dx}{dt}$ **[M1]**

When $x = 1$, $\frac{dy}{dt} = \frac{1}{4}$.

$\frac{1}{4} = -\frac{7}{e^3} \times \frac{dx}{dt}$ **[M1]**

$\frac{dx}{dt} = -\frac{e^3}{28}$ **[A1]**, (must have negative)

$\therefore x$ is decreasing at a rate of $\frac{e^3}{28}$ units/s when $x = 1$.

(Accept rate of change is $-\frac{e^3}{28}$ units/s)

**6** **(i)** $f(x) = 2x^3 + x^2 - 8x - 4$

$= x^2(2x+1) - 4(2x+1)$ **[M1]**

$= (2x+1)(x^2-4)$ **[M1]**

$= \underline{\underline{(2x+1)(x+2)(x-2)}}$ **[A1]**

Alternatively,

$f(x) = 2x^3 + x^2 - 8x - 4$

$f(2) = 2(2)^3 + 2^2 - 8(2) - 4$

$= 0$

$\therefore x - 2$ is a factor of $f(x)$. **[M1]**

Let $2x^3 + x^2 - 8x - 4 = (x-2)(2x^2 + hx + 2)$

Coefficient of $x = -8$.

$2 - 2h = -8$

$h = 5$

$\therefore 2x^3 + x^2 - 8x - 4 = (x-2)(2x^2 + 5x + 2)$ **[M1]**

$= (x-2)(x+2)(2x+1)$

$\therefore f(x) = \underline{\underline{(x-2)(x+2)(2x+1)}}$ **[A1]**

**(ii)** $f(x) + 10x + 5 = 0$

$x^2(2x+1) - 4(2x+1) + 5(2x+1) = 0$ **[M1]**

$(2x+1)(x^2+1) = 0$

Since $x^2 + 1 > 0$, $2x + 1 = 0$. **[M1]**

$\therefore$ The equation has only one solution and the value is $\underline{\underline{x = -\frac{1}{2}}}$. **[A1]**

**(iii)** $y = f(x) + kx$

$= 2x^3 + x^2 - 8x - 4 + kx$

$\frac{dy}{dx} = 6x^2 + 2x - 8 + k$ **[M1]**

At the stationary point, $\frac{dy}{dx} = 0$.

$6x^2 + 2x - 8 + k = 0$ **[M1]**

Since the curve has two stationary points, the equation has real and distinct roots. Discriminant > 0.

$2^2 - 4(6)(-8+k) > 0$ **[M1]**

$k - 8 < \frac{1}{6}$

$\underline{\underline{k < 8\frac{1}{6}}}$ **[A1]**

**7** **(i)** $y = 3x^2 - 8x + 6$ …………(1)

$y = x$ ………………….(2)

$3x^2 - 8x + 6 = x$

$3x^2 - 9x + 6 = 0$

$x^2 - 3x + 2 = 0$ **[M1]**

$(x-1)(x-2) = 0$

$x = 1$ or 2 **[A1]**

$x$-coordinates of $A$ and $B$ are 2 and 1 respectively. [A1]

**(ii)** Area of the region bounded by the curve $y = 3x^2 - 8x + 6$ and the line $y = x$ is $\int_1^2 x - (3x^2 - 8x + 6)\,dx$ **[M1]**

$= \left[-x^3 + \frac{9x^2}{2} - 6x\right]_1^2$ **[M1]**

$= -8 + 18 - 12 - \left(-1 + \frac{9}{2} - 6\right)$

$= \frac{1}{2}$ units$^2$ **[A1]**

**(iii)** The curve $x = 3y^2 - 8y + 6$ is a **reflection (or mirror image)** of the curve $y = 3x^2 - 8x + 6$ in the line $y = x$. $\therefore$ the area bounded by the curve $x = 3y^2 - 8y + 6$ and the line $y = x$ is also $\frac{1}{2}$ units$^2$ **[B2], (**1m for the description and 1m for correct answer)

**8** **(i)** $x = 5\cos 2t - 6\sin t$

$\frac{dx}{dt} = -10\sin 2t - 6\cos t$ **[M1]**

When $t = \frac{\pi}{6}$,

$\frac{dx}{dt} = -10\sin\frac{\pi}{3} - 6\cos\frac{\pi}{6}$ **[M1]**

$= -5\sqrt{3} - 3\sqrt{3}$

$= -8\sqrt{3}$ **[A1] (**Do not give the A1 here if they give 13.9)

When $t = \frac{\pi}{6}$, velocity of $P$ is $-8\sqrt{3}$ cm/s

**(ii)** When $P$ is instantaneously at rest, $\frac{dx}{dt}=0$.

$-10\sin 2t-6\cos t=0$

$5(2\sin t\cos t)+3\cos t=0$ **[M1]**

$\cos t(10\sin t+3)=0$

$\cos t=0$ or $\sin t=-\frac{3}{10}$ **[M1]**

Since $0<t<\pi$, $\sin t\neq-\frac{3}{10}$. **[M1]**

When $\cos t=0, t=\underline{\underline{\frac{\pi}{2}}}$ **[A1]**

**(iii)** When $t=0$,

$x=5$ **[M1]**

When $t=\frac{\pi}{2}$,

$x=5\cos\pi-6\sin\frac{\pi}{2}$

$=-11$ **[M1]**

When $t=2$,

$x=5\cos 4-6\sin 2$

$=-8.7240$ **[M1]**

Distance travelled in the first 2 seconds $= 5-(-11)-8.7240-(-11)$ cm

$= 18.276$ cm

$= \underline{\underline{18.3\text{ cm}}}$ (3 sf) **[A1]**

Alternatively, they can use integration.
Distance travelled in the first 2 seconds

$= -\int_0^{\frac{\pi}{2}}-10\sin 2t-6\cos t\,dt+\int_{\frac{\pi}{2}}^{2}-10\sin 2t-6\cos t\,dt$

$=-[5\cos 2t-6\sin t]_0^{\frac{\pi}{2}}+[5\cos 2t-6\sin t]_{\frac{\pi}{2}}^{2}$

$=-\left(5\cos\pi-6\sin\frac{\pi}{2}+5\cos 0-6\sin 0\right)+\left(5\cos 4-6\sin 2-5\cos\pi+6\sin\frac{\pi}{2}\right)$

$=27+5\cos 4-6\sin 2$

$=\underline{\underline{18.276\text{ cm}}}$

**9** **(i)** $A(-3,1)$, $B(-2,6)$.

Gradient of $AB = \frac{6-1}{-2+3}$

$= 5$ **[M1]**

Midpoint of $AB = \left(\frac{-3-2}{2}, \frac{1+6}{2}\right)$

$= \left(-\frac{5}{2}, \frac{7}{2}\right)$ **[M1]**

Equation of **perpendicular bisector** of $AB$ is $\frac{y-\frac{7}{2}}{x+\frac{5}{2}} = -\frac{1}{5}$

i.e. $y = -\frac{1}{5}x + 3$ **[A1]**

**(ii)** Gradient of the line $2x - 3y + 9 = 0$ is $\frac{2}{3}$.

Gradient of $AG$ is $-\frac{3}{2}$. **[M1]**

Equation of $AG$ is $\frac{y-1}{x+3} = -\frac{3}{2}$

i.e. $y = -\frac{3}{2}x - \frac{7}{2}$ **[M1]**

Since $G$ is the intersection of the perpendicular bisector of $AB$ and the line segment $AG$,

$-\frac{1}{5}x + 3 = -\frac{3}{2}x - \frac{7}{2}$ **[M1]**

$\frac{13}{10}x = -\frac{13}{2}$

$x = -5$

When $x = -5$, $y = 4$.

Coordinates of $G = (-5, 4)$ **[A1]**

**(iii)** $AG = \sqrt{(-5+3)^2 + (4-1)^2}$

$= \sqrt{4+9}$

$= \sqrt{13}$ units **[M1]**

Equation of the circle $C_1$ is $(x+5)^2 + (y-4)^2 = 13$ **[A1]**

i.e. $x^2 + y^2 + 10x - 8y + 28 = 0$

**(iv)** Let coordinates of $H$ be $(a, b)$.

$\left(\frac{-5+a}{2}, \frac{4+b}{2}\right) = (-3, 1)$ **[M1]**

$a = -1, b = -2$

Coordinates of $H = (-1, -2)$. **[A1]**

**(v)** Equation of circle $C_2$ is $(x+1)^2 + (y+2)^2 = 13$ **[A1]**

i.e. $x^2 + y^2 + 2x + 4y - 8 = 0$

**10** **(a)** **(i)** $\sqrt{3-e^x}+1-ke^x=0$

$\sqrt{3-e^x}+1=ke^x$

If $k<0$, $ke^x<0$. **[M1]**

But $\sqrt{3-e^x}\geq 0$, $\sqrt{3-e^x}+1>0$ **[M1]**

$\therefore \sqrt{3-e^x}+1\neq ke^x$, i.e. $\sqrt{3-e^x}+1-ke^x\neq 0$ **[M1]**

$\therefore \sqrt{3-e^x}+1-ke^x=0$ has no solution.

**(ii)** $3-\sqrt{3-e^x}=e^x$

$3-e^x=\sqrt{3-e^x}$

$9-6e^x+(e^x)^2=3-e^x$

$(e^x)^2-5e^x+6=0$ **[M1]**

$(e^x-3)(e^x-2)=0$

$e^x=3$ or $2$ **[M1]**

$x=\underline{\underline{\ln 3 \text{ or } \ln 2}}$ **[A1]**

**(b)** **(i)** $\ln\left(\frac{ax}{1-x}\right)=bt$

When $t=0$, $x=\frac{1}{5}$.

$\ln\left(\frac{\frac{a}{5}}{1-\frac{1}{5}}\right)=0$ **[M1]**

$\frac{a}{5}\times\frac{5}{4}=1$

$a=4$ **[A1]**

When $t=1$, $x=\frac{1}{3}$.

$\ln\left(\frac{4\times\frac{1}{3}}{1-\frac{1}{3}}\right)=b$ **[M1]**

$b=\ln\left(\frac{4}{3}\times\frac{3}{2}\right)$

$b=\ln 2$ **[A1]**

$\therefore \underline{\underline{a=4, b=\ln 2}}$

**(ii)** $\ln\left(\frac{4x}{1-x}\right)=t\ln 2$

$\left(\frac{4x}{1-x}\right)=2^t$ **[M1]**

$4x=2^t(1-x)$

$x(4+2^t)=2^t$

$\underline{\underline{x=\frac{2^t}{4+2^t}}}$ **[A1]**

**11** **(i)** $V = V_0 a^t$

$\lg V = \lg V_0 + \lg a^t$

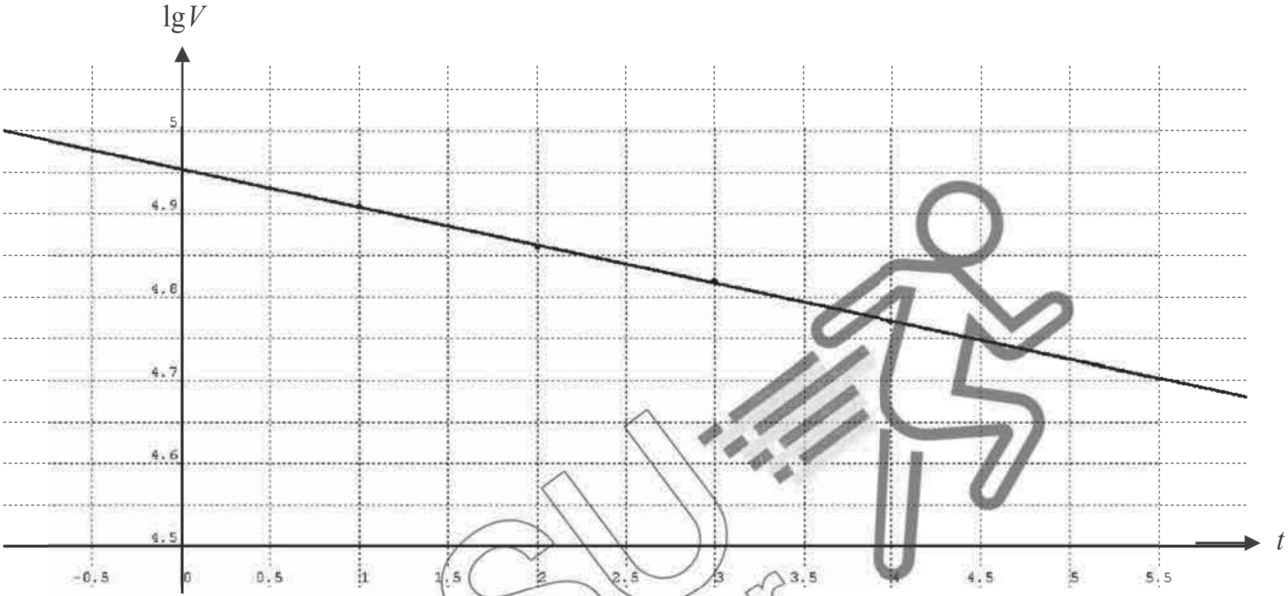
$\lg V = (\lg a)t + \lg V_0$ **[M1]**

| $t$ | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| $\lg V$ | 4.91 | 4.86 | 4.82 | 4.77 |

**Table [M1]**

Graph of $\lg V$ against $t$.

- **All points correctly plotted [M1]**
- **Line cutting the vertical axis [M1]**




**(ii)** $\lg a = -0.04575$ **[M1]**

$a = 10^{-0.04575}$

$= 0.900015$ **[A1]**

$\underline{\underline{a = 0.900\ (3sf)}}$

$\lg V_0 = 4.955$ **[M1]**

$V_0 = 10^{4.955}$

$= 90157$

$\underline{\underline{V_0 = 90100\ (3sf)}}$ **[A1]**

Mr Lee paid \$90100 for the car. **[A1]**

**(iii)** Value of car on 1st January 2018 $= \$90157(0.900)^5$

$= \$53236$

$= \underline{\underline{\$53200\ (3sf)}}$ **[A1]**

End of Paper