Secondary 4E5N
Preliminary Examination 2017
ADDITIONAL MATHEMATICS
Answer Keys for Paper 2

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1 **(i)** $p = -\frac{2}{3}$ or 2

(ii) 8

2 **(i)** Proof

(ii) $x = \frac{\pi}{6}, \frac{5\pi}{6}, \pi, \frac{7\pi}{6}$ or $\frac{11\pi}{6}$

3 **(i)** $\alpha\beta = 2$

(ii) $\alpha + \beta = -4$

(iii) $\alpha - \beta = -\sqrt{8}$
$(x+4)(x+2\sqrt{2}) = 0$

4 **(i)** Use corresponding $\angle$s, $PD \mathbin{//} BC$ **AND** $\angle$s in alternate segment.

(ii) Use the result of (i).

(iii) Use $\angle$s in the same segment, alternate $\angle$s, $PD \mathbin{//} BC$ **AND** $\angle$s in alternate segment.

5 **(i)** $-\frac{6x+1}{e^{3x}}$

(ii) $x > -\frac{1}{6}$

(iii) $-\frac{e^3}{28}$ units/s

6 **(i)** $(2x+1)(x+2)(x-2)$

(ii) $(2x+1)(x^2+1) = 0$
Since $x^2 + 1 > 0,\ 2x + 1 = 0$.
$\therefore$ The equation has only one solution i.e. $x = -\frac{1}{2}$.

(iii) $k < 8\frac{1}{6}$

7 **(i)** x-coordinates of A and B are 2 and 1 respectively.

(ii) $\frac{1}{2}$ units^2

(iii) The curve $x = 3y^2 - 8y + 6$ is a **reflection (or mirror image)** of the curve $y = 3x^2 - 8x + 6$ in the line $y = x$. $\therefore$ the area bounded by the curve $x = 3y^2 - 8y + 6$ and the line $y = x$ is also $\frac{1}{2}$ units^2.

8 **(i)** $-8\sqrt{3}$ cm/s

(ii) $t = \frac{\pi}{2}$,

(iii) 18.3 cm

9 **(i)** $y = -\frac{1}{5}x + 3$

(ii) Equation of AG is $y = -\frac{3}{2}x - \frac{7}{2}$

Coordinates of $G = (-5, 4)$

(iii) $x^2 + y^2 + 10x - 8y + 28 = 0$

(iv) Coordinates of $H = (-1, -2)$.

(v) Equation of circle C_2 is $(x+1)^2 + (y+2)^2 = 13$

10 **(a)** **(i)** Show that $\sqrt{3 - e^x} + 1 - ke^x \neq 0$ or $\sqrt{3 - e^x} + 1 \neq ke^x$

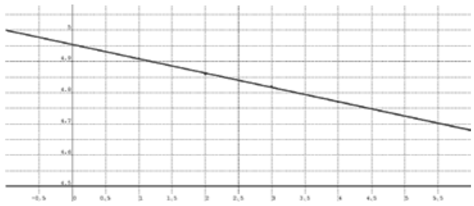
(ii) $\ln 3$ or $\ln 2$

(b) **(i)** $a = 4, b = \ln 2$

(ii) $x = \frac{2^t}{4 + 2^t}$

11 **(i)** $\lg V = (\lg a)t + \lg V_0$

t	1	2	3	4
$\lg V$	4.91	4.86	4.82	4.77



(ii) $a = 0.900$ (3sf), $V_0 = 90100$(3sf). Mr Lee paid \$90100 for the car.

(iii) \$53200

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Paper 2 Marking Scheme

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1 **(i)** $(1-2x)^2(1+px)^7 = (1-4x+4x^2)(1+7px+21p^2x^2+.........)$ **[M1]**

Coefficient of $x^2 = 32$

$21p^2 - 28p + 4 = 32$ **[M1]**

$21p^2 - 28p - 28 = 0$

$3p^2 - 4p - 4 = 0$ **[M1]**

$(3p+2)(p-2) = 0$

$p = -\frac{2}{3}$ or 2 **[A1]**

(ii) Since coefficient of x^2 in $(1-2x)^2(1+px)$ is 32,

Coefficient of x^2 in $(1-x)^2\left(1+\frac{px}{2}\right)^7 = \left(\frac{1}{2}\right)^2 \times 32$ [M1]

$= 8$ [A1]

2 **(i)** $\sin 3x \equiv \sin(2x+x)$,

$\equiv \sin 2x \cos x + \cos 2x \sin x$ **[M1]**

$\equiv 2\sin x \cos^2 x + (1-2\sin^2 x)\sin x$

$\equiv (2\sin x)(1-\sin^2 x) + \sin x - 2\sin^3 x$ **[M1]**

$\equiv 3\sin x - 4\sin^3 x$ [A1]

(ii) $\sin 3x = 2\sin x,\ 0 < x < 2\pi$

$3\sin x - 4\sin^3 x = 2\sin x$

$4\sin^3 x - \sin x = 0$ **[M1]**

$\sin x(4\sin^2 x - 1) = 0$

$\sin x = 0$ or $\pm\frac{1}{2}$ **[M1]**

When $\sin x = 0,\ x = \pi$. **[A1]**

When $\sin x = \frac{1}{2},\ x = \frac{\pi}{6}$ or $\frac{5\pi}{6}$. **[A1]**

When $\sin x = -\frac{1}{2},\ x = \frac{7\pi}{6}$ or $\frac{11\pi}{6}$. **[A1]**

$x = \frac{\pi}{6}, \frac{5\pi}{6}, \pi, \frac{7\pi}{6}$ or $\frac{11\pi}{6}$ (accept $x = 0.524,\ 2.62, 3.14, 3.67$ or 5.76)

3 $x^2 = 12x - 4$

$x^2 - 12x + 4 = 0$

(i) $\alpha^2\beta^2 = 4$ **[M1]**

$\alpha\beta = \pm 2$

Since $\alpha < 0 \,\&\, \beta < 0, \alpha\beta > 0.$

$\therefore \alpha\beta = 2$ **[A1]**

(ii) $\alpha^2 + \beta^2 = 6$ **[M1]**

$(\alpha+\beta)^2 - 2\alpha\beta = 12$

$(\alpha+\beta)^2 = 12 + 2(2)$

$= 16$ **[M1]**

$\alpha + \beta = \pm 4$

Since $\alpha < 0, \beta < 0$, $\alpha + \beta < 0$,

$\therefore \alpha + \beta = -4$ **[A1]**

(iii) $(\alpha - \beta)^2 = \alpha^2 + \beta^2 - 2\alpha\beta$

$= 12 - 2(2)$

$= 8$ **[M1]**

$\alpha - \beta = \pm\sqrt{8}$

Since $\alpha < \beta$, $\alpha - \beta < 0$.

$\therefore \alpha - \beta = -\sqrt{8}$ **[M1]**

Quadratic equation with roots $\alpha + \beta$ and $\alpha - \beta$ is $(x+4)(x+2\sqrt{2}) = 0$ **[A1]**

4 **(i)** $\angle ADP = \angle ACB$ (corresponding $\angle$ s, $PD \parallel BC$) **[M1]**

$= \angle ABP$ ($\angle$ s in alternate segment) **[M1]**

(ii) Since $\angle ADP = \angle ABP$ from (i), using angles in the segment, A, D, B and P lie on a circle. **[M1]**

(iii) $\angle BAP = \angle BDP$ ($\angle$ s in the same segment) **[M1]**

$= \angle DBC$ (alternate $\angle$ s, $PD \parallel BC$) **[M1]**

$\angle BAP = \angle BCD$ ($\angle$ s in alternate segment) **[M1]**

Since $\angle DBC = \angle BCD$,

$\therefore DB = DC$ **[A1]**

5 $y = \frac{1+2x}{e^{3x}}$

(i) $\frac{dy}{dx} = \frac{e^{3x}(2) - (1+2x)(3e^{3x})}{e^{6x}}$ **[M1]**

$= \frac{2-3-6x}{e^{3x}}$

$= -\frac{6x+1}{e^{3x}}$ **[A1]** (accept $\frac{-6x-1}{e^{3x}}$)

(ii) For y to be decreasing, $\frac{dy}{dx} < 0$.

$-\frac{6x+1}{e^{3x}} < 0$ **[M1]**

Since $e^{3x} > 0$, **[M1]**

$-(6x+1) < 0$.

$x > -\frac{1}{6}$ **[A1]**

(iii) $\frac{dy}{dt} = \frac{dy}{dx} \times \frac{dx}{dt}$

$= -\frac{6x+1}{e^{3x}} \times \frac{dx}{dt}$ **[M1]**

When $x = 1$, $\frac{dy}{dt} = \frac{1}{4}$.

$\frac{1}{4} = -\frac{7}{e^3} \times \frac{dx}{dt}$ **[M1]**

$\frac{dx}{dt} = -\frac{e^3}{28}$ **[A1]**, (must have negative)

$\therefore x$ is decreasing at a rate of $\frac{e^3}{28}$ units/s when $x = 1$.

(Accept rate of change is $-\frac{e^3}{28}$ units/s)

6 **(i)** $f(x) = 2x^3 + x^2 - 8x - 4$

$= x^2(2x+1) - 4(2x+1)$ **[M1]**

$= (2x+1)(x^2-4)$ **[M1]**

$= \underline{\underline{(2x+1)(x+2)(x-2)}}$ **[A1]**

Alternatively,

$f(x) = 2x^3 + x^2 - 8x - 4$

$f(2) = 2(2)^3 + 2^2 - 8(2) - 4$

$= 0$

$\therefore x-2$ is a factor of $f(x)$. **[M1]**

Let $2x^3 + x^2 - 8x - 4 = (x-2)(2x^2 + hx + 2)$

Coefficient of $x = -8$.

$2 - 2h = -8$

$h = 5$

$\therefore 2x^3 + x^2 - 8x - 4 = (x-2)(2x^2 + 5x + 2)$ **[M1]**

$= (x-2)(x+2)(2x+1)$

$\therefore f(x) = \underline{\underline{(x-2)(x+2)(2x+1)}}$ **[A1]**

(ii) $f(x) + 10x + 5 = 0$

$x^2(2x+1) - 4(2x+1) + 5(2x+1) = 0$ **[M1]**

$(2x+1)(x^2+1) = 0$

Since $x^2 + 1 > 0,\ 2x+1 = 0$. **[M1]**

$\therefore$ The equation has only one solution and the value is $\underline{\underline{x = -\frac{1}{2}}}$. **[A1]**

(iii) $y = f(x) + kx$

$= 2x^3 + x^2 - 8x - 4 + kx$

$\frac{dy}{dx} = 6x^2 + 2x - 8 + k$ **[M1]**

At the stationary point, $\frac{dy}{dx} = 0$.

$6x^2 + 2x - 8 + k = 0$ **[M1]**

Since the curve has two stationary points, the equation has real and distinct roots. Discriminant > 0.

$2^2 - 4(6)(-8+k) > 0$ **[M1]**

$k - 8 < \frac{1}{6}$

$\underline{\underline{k < 8\frac{1}{6}}}$ **[A1]**

7 **(i)** $y = 3x^2 - 8x + 6$ ………..(1)

$y = x$ …………………(2)

$3x^2 - 8x + 6 = x$

$3x^2 - 9x + 6 = 0$

$x^2 - 3x + 2 = 0$ **[M1]**

$(x-1)(x-2) = 0$

$x = 1$ or 2 **[A1]**

x-coordinates of A and B are 2 and 1 respectively. [A1]

(ii) Area of the region bounded by the curve $y = 3x^2 - 8x + 6$ and the line $y = x$ is $\int_1^2 x - (3x^2 - 8x + 6)\,dx$ **[M1]**

$= \left[-x^3 + \frac{9x^2}{2} - 6x\right]_1^2$ **[M1]**

$= -8 + 18 - 12 - \left(-1 + \frac{9}{2} - 6\right)$

$= \frac{1}{2}$ units^2 **[A1]**

(iii) The curve $x = 3y^2 - 8y + 6$ is a **reflection (or mirror image)** of the curve $y = 3x^2 - 8x + 6$ in the line $y = x$. $\therefore$ the area bounded by the curve $x = 3y^2 - 8y + 6$ and the line $y = x$ is also $\frac{1}{2}$ units^2 **[B2], (**1m for the description and 1m for correct answer)

8 **(i)** $x = 5\cos 2t - 6\sin t$

$\frac{dx}{dt} = -10\sin 2t - 6\cos t$ **[M1]**

When $t = \frac{\pi}{6}$,

$\frac{dx}{dt} = -10\sin\frac{\pi}{3} - 6\cos\frac{\pi}{6}$ **[M1]**

$= -5\sqrt{3} - 3\sqrt{3}$

$= -8\sqrt{3}$ **[A1] (**Do not give the A1 here if they give 13.9)

When $t = \frac{\pi}{6}$, velocity of P is $-8\sqrt{3}$ cm/s

(ii) When P is instantaneously at rest, $\frac{dx}{dt} = 0$.

$-10\sin 2t - 6\cos t = 0$

$5(2\sin t\cos t) + 3\cos t = 0$ **[M1]**

$\cos t(10\sin t + 3) = 0$

$\cos t = 0$ or $\sin t = -\frac{3}{10}$ **[M1]**

Since $0 < t < \pi$, $\sin t \neq -\frac{3}{10}$. **[M1]**

When $\cos t = 0, t = \underline{\underline{\frac{\pi}{2}}}$ **[A1]**

(iii) When $t = 0$,

$x = 5$ **[M1]**

When $t = \frac{\pi}{2}$,

$x = 5\cos\pi - 6\sin\frac{\pi}{2}$

$= -11$ **[M1]**

When $t = 2$,

$x = 5\cos 4 - 6\sin 2$

$= -8.7240$ **[M1]**

Distance travelled in the first 2 seconds $= 5 - (-11) - 8.7240 - (-11)$ cm

$= 18.276$ cm

$= \underline{\underline{18.3\text{ cm}}}$ (3 sf) **[A1]**

Alternatively, they can use integration.
Distance travelled in the first 2 seconds

$= -\int_0^{\frac{\pi}{2}} -10\sin 2t - 6\cos t \, dt + \int_{\frac{\pi}{2}}^{2} -10\sin 2t - 6\cos t \, dt$

$= -[5\cos 2t - 6\sin t]_0^{\frac{\pi}{2}} + [5\cos 2t - 6\sin t]_{\frac{\pi}{2}}^{2}$

$= -\left(5\cos\pi - 6\sin\frac{\pi}{2} + 5\cos 0 - 6\sin 0\right) + \left(5\cos 4 - 6\sin 2 - 5\cos\pi + 6\sin\frac{\pi}{2}\right)$

$= 27 + 5\cos 4 - 6\sin 2$

$= \underline{\underline{18.276\text{ cm}}}$

9 **(i)** $A(-3,1)$, $B(-2,6)$.

Gradient of $AB = \frac{6-1}{-2+3}$

$= 5$ **[M1]**

Midpoint of $AB = \left(\frac{-3-2}{2}, \frac{1+6}{2}\right)$

$= \left(-\frac{5}{2}, \frac{7}{2}\right)$ **[M1]**

Equation of **perpendicular bisector** of AB is $\frac{y-\frac{7}{2}}{x+\frac{5}{2}} = -\frac{1}{5}$

i.e. $y = -\frac{1}{5}x + 3$ **[A1]**

(ii) Gradient of the line $2x - 3y + 9 = 0$ is $\frac{2}{3}$.

Gradient of AG is $-\frac{3}{2}$. **[M1]**

Equation of AG is $\frac{y-1}{x+3} = -\frac{3}{2}$

i.e. $y = -\frac{3}{2}x - \frac{7}{2}$ **[M1]**

Since G is the intersection of the perpendicular bisector of AB and the line segment AG,

$-\frac{1}{5}x + 3 = -\frac{3}{2}x - \frac{7}{2}$ **[M1]**

$\frac{13}{10}x = -\frac{13}{2}$

$x = -5$

When $x = -5$, $y = 4$.

Coordinates of $G = (-5, 4)$ **[A1]**

(iii) $AG = \sqrt{(-5+3)^2 + (4-1)^2}$

$= \sqrt{4+9}$

$= \sqrt{13}$ units **[M1]**

Equation of the circle C_1 is $(x+5)^2 + (y-4)^2 = 13$ **[A1]**

i.e. $x^2 + y^2 + 10x - 8y + 28 = 0$

(iv) Let coordinates of H be (a,b).

$\left(\frac{-5+a}{2}, \frac{4+b}{2}\right) = (-3,1)$ **[M1]**

$a = -1, b = -2$

Coordinates of $H = (-1,-2)$. **[A1]**

(v) Equation of circle C_2 is $(x+1)^2 + (y+2)^2 = 13$ **[A1]**

i.e. $x^2 + y^2 + 2x + 4y - 8 = 0$

10 (a) (i) $\sqrt{3-e^x}+1-ke^x=0$

$\sqrt{3-e^x}+1=ke^x$

If $k<0$, $ke^x<0$. **[M1]**

But $\sqrt{3-e^x}\geq 0$, $\sqrt{3-e^x}+1>0$ **[M1]**

$\therefore \sqrt{3-e^x}+1\neq ke^x$, i.e. $\sqrt{3-e^x}+1-ke^x\neq 0$ **[M1]**

$\therefore \sqrt{3-e^x}+1-ke^x=0$ has no solution.

(ii) $3-\sqrt{3-e^x}=e^x$

$3-e^x=\sqrt{3-e^x}$

$9-6e^x+(e^x)^2=3-e^x$

$(e^x)^2-5e^x+6=0$ **[M1]**

$(e^x-3)(e^x-2)=0$

$e^x=3 \text{ or } 2$ **[M1]**

$x=\underline{\underline{\ln 3 \text{ or } \ln 2}}$ **[A1]**

(b) (i) $\ln\left(\frac{ax}{1-x}\right)=bt$

When $t=0$, $x=\frac{1}{5}$.

$\ln\left(\frac{\frac{a}{5}}{1-\frac{1}{5}}\right)=0$ **[M1]**

$\frac{a}{5}\times\frac{5}{4}=1$

$a=4$ **[A1]**

When $t=1$, $x=\frac{1}{3}$.

$\ln\left(\frac{4\times\frac{1}{3}}{1-\frac{1}{3}}\right)=b$ **[M1]**

$b=\ln\left(\frac{4}{3}\times\frac{3}{2}\right)$

$b=\ln 2$ **[A1]**

$\therefore \underline{\underline{a=4, b=\ln 2}}$

(ii) $\ln\left(\frac{4x}{1-x}\right)=t\ln 2$

$\left(\frac{4x}{1-x}\right)=2^t$ **[M1]**

$4x=2^t(1-x)$

$x(4+2^t)=2^t$

$\underline{\underline{x=\frac{2^t}{4+2^t}}}$ **[A1]**

11 **(i)** $V = V_0 a^t$

$\lg V = \lg V_0 + \lg a^t$

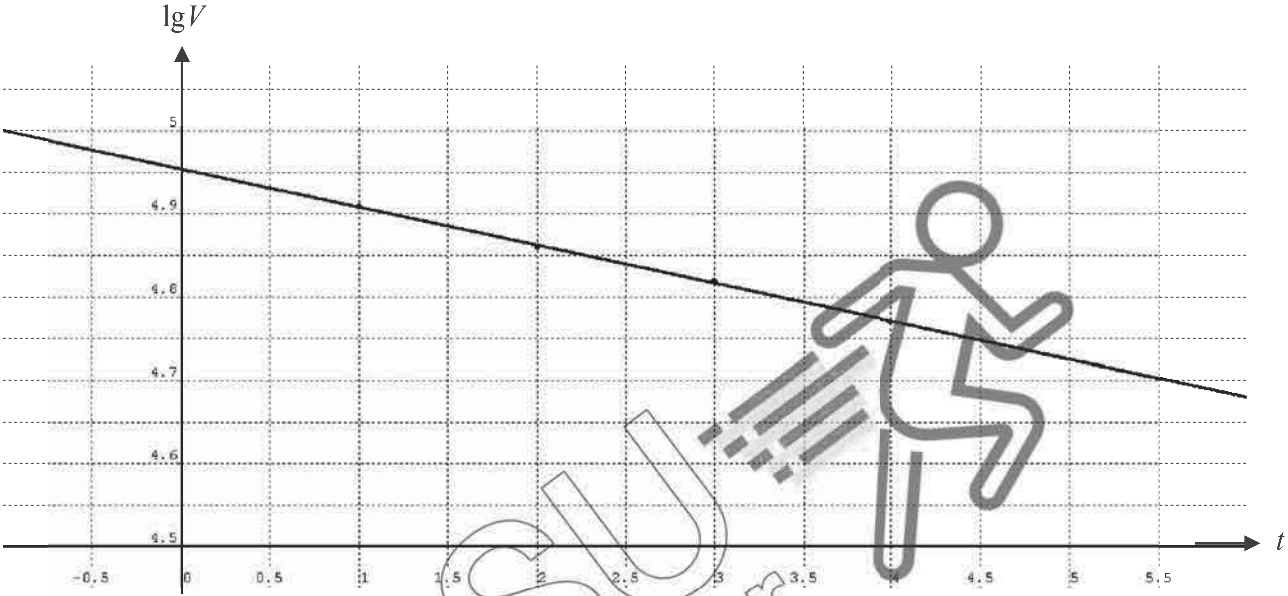
$\lg V = (\lg a)t + \lg V_0$ **[M1]**

t	1	2	3	4
$\lg V$	4.91	4.86	4.82	4.77

Table [M1]

Graph of $\lg V$ against *t*.

- **All points correctly plotted [M1]**
- **Line cutting the vertical axis [M1]**



(ii) $\lg a = -0.04575$ **[M1]**

$a = 10^{-0.04575}$

$= 0.900015$ **[A1]**

$\underline{\underline{a = 0.900 \text{ (3sf)}}}$

$\lg V_0 = 4.955$ **[M1]**

$V_0 = 10^{4.955}$

$= 90157$

$\underline{\underline{V_0 = 90100 \text{ (3sf)}}}$ **[A1]**

Mr Lee paid \$90100 for the car. **[A1]**

(iii) Value of car on 1st January 2018 $= \$90157(0.900)^5$

$= \$53236$

$= \underline{\underline{\$53200 \text{ (3sf)}}}$ **[A1]**

End of Paper