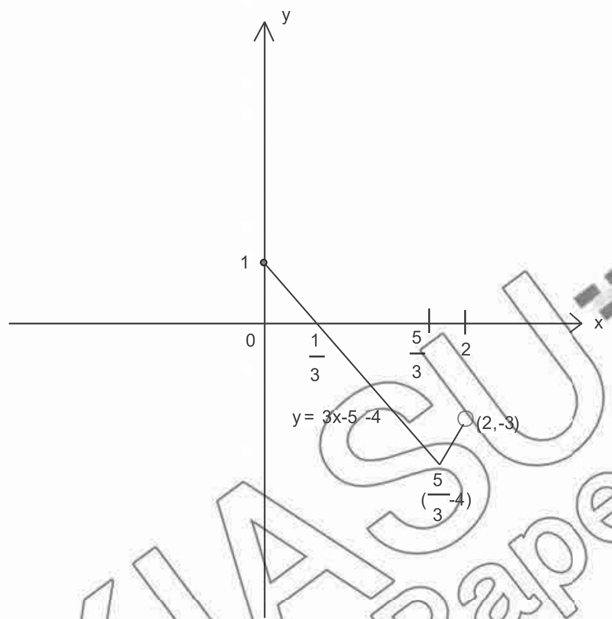
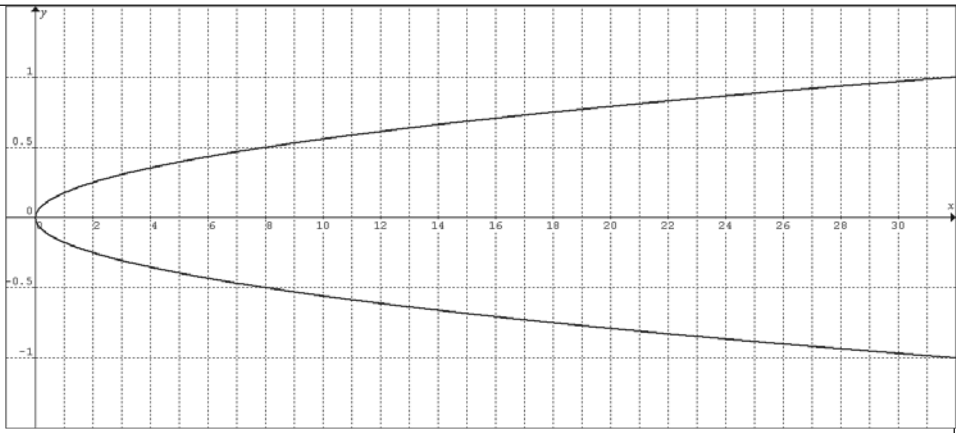
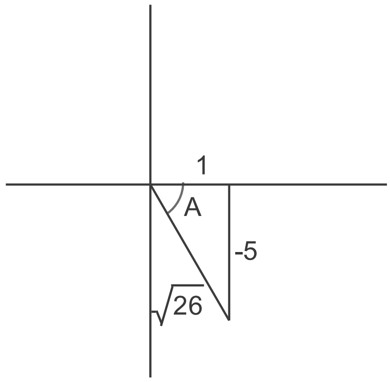
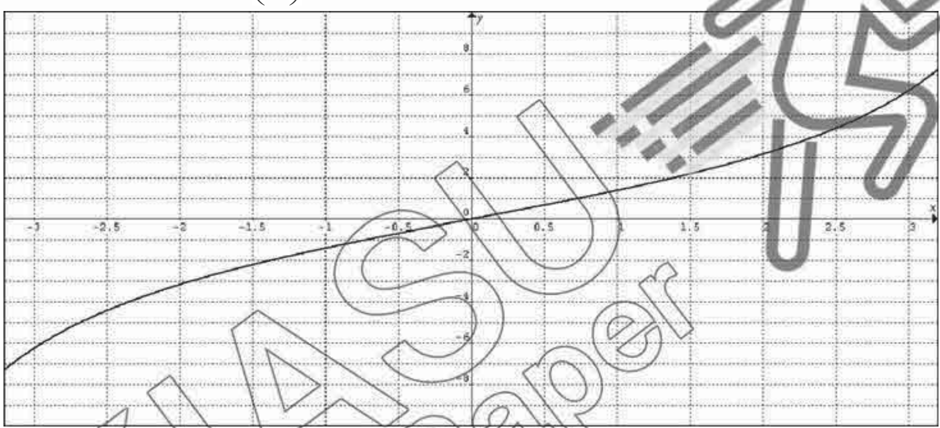
1	The line $x + y = h$, where h is a constant, is tangent to the curve $y = 3x^2 + 5x$ at the point R. Find the value of h and the coordinates of R.	[5]
	Q1 Solution (M): Substitute $y = h - x$ into $y = 3x^2 + 5x$ $h - x = 3x^2 + 5x$ $3x^2 + 6x - h = 0$ For tangent, $b^2 - 4ac = 0$ $36 - 4\times 3(-h) = 0$ $12h + 36 = 0$ $h = -3$ Substitute $h = -3$ into $3x^2 + 6x - h = 0$ to find x – coordinate of R $3x^2 + 6x + 3 = 0$ $x^2 + 2x + 1 = 0$ $(x+1)^2 = 0$ $x = -1$ Substitute $x = -1$ into $y = 3x^2 + 5x$ $y = 3 - 5 = -2$ Coordinates of R are $(-1, -2)$ Alternative From $y + x = h$, Gradient $= -1$ $\frac{dy}{dx} = -1$ $\frac{dy}{dx} = 6x + 5$ $6x + 5 = -1$ $x = -1$ Substitute $x = -1$ into $y = 3x^2 + 5x$ $y = 3 - 5 = -2$ Coordinates of R are $(-1, -2)$ Substitute $(-1, -2)$ into $y + x = h$ $h = -3$	M 1 M 1 A1 M 1 A1 M 1 M 1 M 1 A1 A1

2	Without using a calculator, find the values of the integers m and n for which the solution of the equation $x\sqrt{12} = x\sqrt{\frac{2}{27}} + \sqrt{108}$ is $\frac{m+n\sqrt{2}}{161}$.	[5]
	Solutions for Q2(M) $x\sqrt{12} = x\sqrt{\frac{2}{27}} + \sqrt{108}$ $x\left(\sqrt{12} - \sqrt{\frac{2}{27}}\right) = \sqrt{108}$ $x\left(2\sqrt{3} - \frac{\sqrt{2}}{3\sqrt{3}}\right) = 6\sqrt{3}$ $x\left(\frac{2\sqrt{3}\times 3\sqrt{3} - \sqrt{2}}{3\sqrt{3}}\right) = 6\sqrt{3}$ $x\left(\frac{18-\sqrt{2}}{3\sqrt{3}}\right) = 6\sqrt{3}$ $x = \frac{6\sqrt{3}\times 3\sqrt{3}}{18-\sqrt{2}}$ $x = \frac{6\sqrt{3}\times 3\sqrt{3}}{18-\sqrt{2}} \times \frac{18+\sqrt{2}}{18+\sqrt{2}}$ $x = \frac{54(18+\sqrt{2})}{324-2}$ $x = \frac{27(18+\sqrt{2})}{161}$ $x = \frac{486+27\sqrt{2}}{161}$ Therefore, $m = 486, n = 27$ Alternatively, Substitute $x = \frac{m+n\sqrt{2}}{161}$ into eqn to get $\frac{\sqrt{12}m + n\sqrt{24}}{161} = \frac{\sqrt{2}m}{161} + \frac{2n}{\sqrt{27}(161)} + \sqrt{108}$ $\frac{2\sqrt{3}}{161}m + \frac{2\sqrt{6}}{161}n = \frac{\sqrt{2}}{3\sqrt{3}(161)}m + \frac{2n}{3\sqrt{3(161)}} + 6\sqrt{3}$ $\frac{6}{161}m + \frac{6\sqrt{2}}{161}n = \frac{\sqrt{2}}{3(161)}m + \frac{2n}{3(161)} + 18$ *By comparing surds* $18m = 2n + 8964, \quad 18(18n) = 2n + 8694$ $18n = m \qquad n = 27$ $m = 486$	M 1 M 1 M 1 M 1 A1 M1 M1 M1 M1 A1

3	Express $\dfrac{x^2+5}{(x^2-1)(x+1)}$ in partial fractions.	[5]
	Q3 Solution (M) $\dfrac{x^2+5}{(x^2-1)(x+1)} = \dfrac{x^2+5}{(x-1)(x+1)(x+1)}$ $= \dfrac{x^2+5}{(x-1)(x+1)^2}$ $= \dfrac{A}{x-1} + \dfrac{B}{x+1} + \dfrac{C}{(x+1)^2}$ $x^2+5 = A(x+1)^2 + B(x-1)(x+1) + C(x-1)$ let $x=1$ $1+5=4A$ $A=\dfrac{3}{2}$ Let $x=-1$ $1+5=-2C$ $C=-3$ Let $x=0$ $5=\dfrac{3}{2}-B+3$ $B=-\dfrac{1}{2}$ Hence $\dfrac{x^2+5}{(x^2-1)(x+1)} = \dfrac{3}{2(x-1)} - \dfrac{1}{2(x+1)} - \dfrac{3}{(x+1)^2}$	M1 M1 M2 For all 3 corr A1
4	(a) The graph of $y=\|3x+q\|$ passes through the point (-2,5), find the possible values of q. (b) (i) Solve the inequality $\|3x-5\|>4$ (ii) Sketch the graph of $y=\|3x-5\|-4$ for $0 \le x < 2$.	[2] [2] [2]
4a	Q4 Solutions (M) Substitute (-2, 5) into $y=\|3x+q\|$	M 1

4b (i) (ii)	$5 = \|3(-2)+q\|$ $5 = -6+q \quad or \quad -5 = -6+q$ $q = 11 \quad or \quad q = 1$ $\|3x-5\| > 4$ $3x-5 > 4 \quad or \quad 3x-5 < -4$ $x > 3 \quad or \quad x < \frac{1}{3}$  	A1 M1 A1 G1 for shape G1 for axes, coord (0,1), (2,-3), $(\frac{1}{3},0)$ $(\frac{5}{3},-4)$ and label
5	(i) Sketch the graph of $y^2 = \frac{x}{32}$ where $x \geq 0$. (ii) Find the x-coordinates of the points of intersection when the curve $y = x^3$ meets the curve $y^2 = \frac{x}{32}$.	[2] [3]
	Q5 Solution (M) Graph of $y^2 = \frac{x}{32}$	G1 for shape G1 for coord, (0,0), (8,0.5)

		(8,-0.5), o.e axes, and label M1
	$y = x^3$ $y^2 = x^6$ $\therefore\ x^6 = \frac{x}{32}$ $x^6 - \frac{x}{32} = 0$ $x\left(x^5 - \frac{1}{32}\right) = 0$ $x = \sqrt[5]{\frac{1}{32}} \quad or \quad x = 0$ $x = \frac{1}{2}\ or\ x = 0$	M1 A1
6(a) 6(b)	Given that $A = \tan^{-1}(-5)$, where A is the principal value, find the exact value of (i) $\cot A$ (ii) $\sec A$ (iii) $\sin(-A)$ Sketch the graph of $y = 4\tan\left(\frac{x}{3}\right)$ where $-\pi \le x \le \pi$.	 [1] [1] [1] [3]
	Q6 Solution (M) A being the principal value means $-\frac{\pi}{2} < A < \frac{\pi}{2}$ for tangent function. Note: principal values for sine is $-\frac{\pi}{2} \le A \le \frac{\pi}{2}$. For cosine is $0 \le A \le \pi$.	 B1

<table>
<tr><td></td><td>$(i)\quad A=\tan^{-1}(-5)$
$\tan A=-5$
$\cot A=-\frac{1}{5}$
$(ii)\quad \sec A=\sqrt{26}$
$(iii)\quad \sin(-A)=-\sin A$
$=-\left(\frac{-5}{\sqrt{26}}\right)$
$=\frac{5}{\sqrt{26}}$

Solution for 6b
Graph of $y=4\tan\left(\frac{x}{3}\right)$
</td><td>B1

B1

G1 for shape

G2 for axes, label, coord $(\frac{3}{4}\pi,4)$, $(-\frac{3}{4}\pi,-4)$ $(\pi,4\sqrt{3})$, $(-\pi,-4\sqrt{3})$

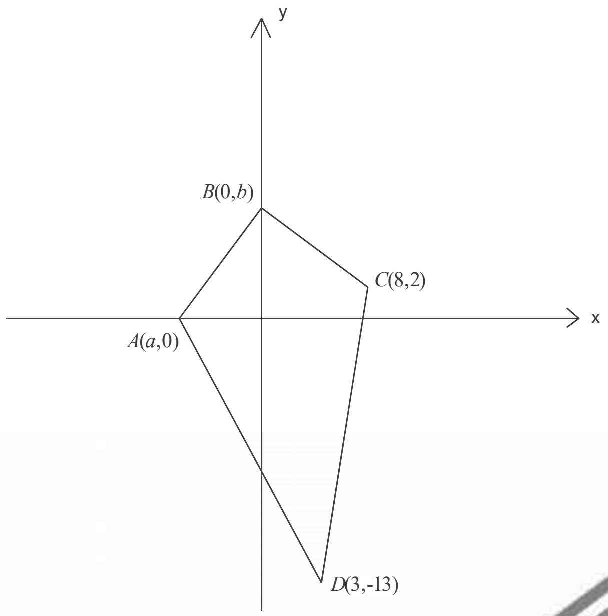
G1 for at least 2 coord correct</td></tr>
<tr><td>7</td><td>(a) Prove that $\sec x\,\mathrm{cosec}\,x=\cot x+\tan x$.

(b) Solve the equation $\cos^2 y-\sin^2 y=4\sin y\cos y$ for $-180°\le y\le 180°$.</td><td>[3]

[5]</td></tr>
<tr><td>7</td><td>Solution
(a)</td><td>M1</td></tr>
</table>

	$RHS = \cot x + \tan x$	
	$= \dfrac{\cos x}{\sin x} + \dfrac{\sin x}{\cos x}$	M1
	$= \dfrac{\cos^2 x + \sin^2 x}{\sin x \cos x}$	M1
	$= \dfrac{1}{\sin x \cos x}$	
	$= \dfrac{1}{\cos x} \cdot \dfrac{1}{\sin x}$	
	$= \sec x \operatorname{cosec} x$	
	$= LHS$	
	(b)	M1
	$\cos^2 y - \sin^2 y = 4\sin y \cos y$	
	$\cos 2y = 2(2\sin y \cos y)$	M1
	$\cos 2y = 2\sin 2y$	
	$\tan 2y = \dfrac{1}{2}$	
	$-360° < 2y < 360°$	M1
	$2y$ ==>1st or 3rd quad	
	Basic angle $= \tan^{-1}\left(\dfrac{1}{2}\right) = 26.565°$	A2
	$2y = 26.565°, 206.565°, -333.435°, -153.435°$	
	$y = 13.283°, 103.283°, -166.718°, -76,718°$	
	y ≈ -76.7°, -166.7°, 13.3°, 103.3°	
	Alternative	
	$\cos^2 y - \sin^2 y = 4\sin y \cos y$	
	$\dfrac{\cos^2 y}{\cos^2 y} - \dfrac{\sin^2 y}{\cos^2 y} = \dfrac{4\sin y \cos y}{\cos^2 y}$	M1
	$1 - \tan^2 y = 4\tan y$	
	$\tan^2 y + 4\tan y - 1 = 0$	M1
	$\tan y = \dfrac{-4 \pm \sqrt{4^2 - 4(1)(-1)}}{2(1)}$	
	$\tan y = 0.23607$	M1
	$y = 13.283°, (180° + 13.283°) - 360° = -166.717°$	A1
	$\tan y = -4.236067$	
	$y = -76.717, \quad \text{or } 180° - 76.717° = 103.283°$	A1

	Alternative	
	A small number of students used R-formula $\cos^2 y - \sin^2 y = 4\sin y\cos y$ $\cos 2y = 2\sin 2y$ $2\sin 2y - \cos 2y = 0$ $2\sin 2y - \cos 2y = R\sin(2y - \alpha)$ $= R\sin 2y\cos\alpha - R\cos 2y\sin\alpha$ $R = \sqrt{2^2 + (-1)^2} = \sqrt{5}$ $\tan\alpha = \frac{1}{2}$ $\alpha = 26.565°$ $2\sin 2y - \cos 2y = \sqrt{5}\sin(2y - 26.565°)$ $\sqrt{5}\sin(2y - 26.565°) = 0$ $\sin(2y - 26.565°) = 0$ $-180° < y < 180°$ $-386.565° < 2y - 26.565° < 333.565$ $2y - 26.565° = -360°, -180°, 0°, 180°$ $2y = -333.435°, -153.435°, 26.565°, 206.565°$ $y = -166.7°, -76.7°, 13.3°, 103.3°$	M1 M1 M1 A2
8	The diagram shows a kite *ABCD* in which the coordinates of *C* and *D* are $(8, 2)$ and $(3, -13)$ respectively. Given that the point *B* and *C* lies on the *x*-axis and *y*-axis respectively, find the (i) gradient of *CD*; (ii) coordinates of *A* and of *B*; (iii) midpoint of *AC*; and (iv) area of the kite.	[1] [4] [1] [2]



Q8 Solution (M)

(i) Gradient of $CD = \dfrac{2+13}{8-3}$
$= 3$ B1

(ii) Using distance formula for lines *AD* and *DC*,

$(a-3)^2 + 13^2 = 15^2 + 5^2$ M 1

$(a-3)^2 = 15^2 + 5^2 - 13^2$

$(a-3)^2 = 81$

$a-3 = 9 \quad or \quad a-3 = -9$

$a = 12(n.a.) \; or \quad a = -6$ A1

The coordinates of *A* are (-6,0)

Using distance formula for lines *AB* and *BC,*

$b^2 + 6^2 = (b-2)^2 + (-8)^2$ M1

$b^2 + 36 = b^2 - 4b + 4 + 64$

$4b = 68 - 36$

$b = 8$

The coordinates of *B* are (0,8) A1

(iii) Midpoint of $AC = \left(\dfrac{-6+8}{2}, \dfrac{0+2}{2}\right) = (1,1)$ B1

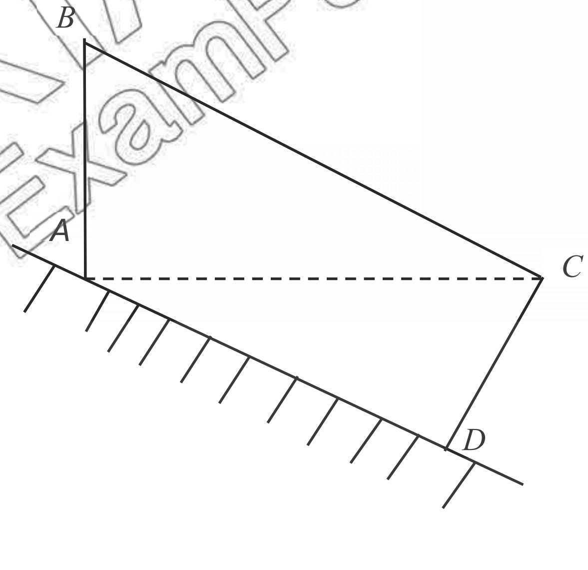
	(iv) Area of kite $= \frac{1}{2}\begin{vmatrix} -6 & 3 & 8 & 0 & -6 \\ 0 & -13 & 2 & 8 & 0 \end{vmatrix}$ $= \frac{1}{2}\lvert(78+6+64)-(-104-48)\rvert$ $= \frac{1}{2}\lvert 148+152\rvert$ $= 150 \text{ units}^2$	M 1 A1
9	(a) Find the equation of the tangent to the curve $y=(2x-1)^3$, for $x > \frac{1}{4}$, which is perpendicular to the line $3y+2x=9$. (b) A curve is such that $\frac{dy}{dx} = 6\cos 2x + 1$ and passes through the point $\left(\frac{\pi}{4}, \frac{\pi}{4}+1\right)$. Find the equation of the curve.	[7] [3]
	$\frac{dy}{dx} = 3(2x-1)^2(2) = 6(2x-1)^2$ For the line $3y+2x=9$ $y = -\frac{2}{3}x+3$ gradient of line $= -\frac{2}{3}$ $m_T m_N = -1$ $m_T = \frac{3}{2}$ When $6(2x-1)^2 = \frac{3}{2}$ $(2x-1)^2 = \frac{1}{4}$ $2x-1 = \frac{1}{2}$ or $2x-1 = -\frac{1}{2}$ $2x = \frac{3}{2}$ or $2x = \frac{1}{2}$ $x = \frac{3}{4}$ or $x = \frac{1}{4}$ (rejected)	M1 M1 M1 M1 A1

	When $x=\frac{3}{4}$, $y=\left(2\cdot\frac{3}{4}-1\right)^3=\frac{1}{8}$ Eqn of tangent is $y-\frac{1}{8}=\frac{3}{2}\left(x-\frac{3}{4}\right)$ $y=\frac{3}{2}x-\frac{9}{8}+\frac{1}{8}$ $y=\frac{3}{2}x-1$	M1 A1
	9b) (i) $y=\int(6\cos 2x+1)dx$ $=\frac{6\sin 2x}{2}+x+c$ $y=3\sin 2x+x+c$ When $x=\frac{\pi}{4}$, $y=\frac{\pi}{4}+1$ $\frac{\pi}{4}+1=3\sin\frac{\pi}{2}+\frac{\pi}{4}+c$ $c=1-3=-2$ $\therefore y=3\sin 2x+x-2$	M1 M1 A1
10 (a)	**(i)** Find $\frac{d}{dx}\left(e^{x^2}\right)$. **(ii)** Evaluate $\int_0^1 xe^{x^2}dx$	[1] [2]
	Q10 Solution (i) $\frac{d}{dx}\left(e^{x^2}\right)=2xe^{x^2}$ (ii) $\int_0^1 2xe^{x^2}dx=\left[e^{x^2}\right]_0^1$ $\int_0^1 xe^{x^2}dx=\frac{1}{2}\left[e^{x^2}\right]_0^1=\frac{1}{2}(e-1)$ or 0.859	B1 M1 A1
10 (b)	Given that $f(x)=\frac{\ln x}{x-1}$ for $x>1$.	

<table>
<tr><td></td><td>(i) Show that $f'(x)=\dfrac{x(1-\ln x)-1}{x(x-1)^2}$.
(ii) Hence, find the equation of the normal to the curve $y=f(x)$ at the point where $x=2$.</td><td>[2]
[3]</td></tr>
<tr><td></td><td>Q10 (b) Solution
(i)
$f'(x)=\dfrac{(x-1)\cdot\frac{1}{x}-(\ln x)(1)}{(x-1)^2}$
$=\dfrac{x-1-x\ln x}{x(x-1)^2}$
$=\dfrac{x(1-\ln x)-1}{x(x-1)^2}$

(ii)
When $x=2$, $f'(2)=-0.193147$
$y=\ln 2=0.693147$
Gradient of normal at $=-1\div(-0.193147)=5.1774$
Eqn of normal is $y-0.693147=5.1774(x-2)$
$y=5.1774x-9.661653$
$y=5.18x-9.66$</td><td>M1
A1

M1
M1
A1</td></tr>
<tr><td>11</td><td>Find the positive number, x, when added to twice its reciprocal gives a minimum sum. [5]</td><td>[5]</td></tr>
<tr><td></td><td>Q11 Solution</td><td>M1
M1
A1</td></tr>
</table>

	Let y be the sum of x and twice its reciprocal. $y = x + \frac{2}{x} = x + 2x^{-1}$ $\frac{dy}{dx} = 1 - \frac{2}{x^2}$ When $\frac{dy}{dx} = 0$ $1 - \frac{2}{x^2} = 0$ $\frac{2}{x^2} = 1$ $x^2 = 2$ $x = \sqrt{2}$ only (or 1.41) since x is positive $\frac{d^2y}{dx^2} = \frac{4}{x^3}$ When $x = \sqrt{2}, \frac{d^2y}{dx^2} = \frac{4}{\sqrt{2}^3} \approx 1.41 (> 0)$ Hence when $x = \sqrt{2}$, sum will be minimum.	 M! A1
12	The diagram shows the plan of a field. On one side of the field is a wall AD. The farmer wants to fence up the field with the solid lines, AB, BC, and CD representing the fencing.  Angles BAC and CDA are right angles, $\angle ACB = \angle DAC = \theta$ radians, and $BC = 50$ m.	

	(i) Show that the total length of fencing, P m, is given by $P = 25\sin 2\theta + 50\sin\theta + 50$. (ii) Determine the stationary value of P . (iii) Give a reason why this value of P is a maximum value.	[2] [5] [2]
	Q12 Solution $(i)\ BA = 50\sin\theta$ $AC = 50\cos\theta$ $CD = AC\sin\theta$ $= 50\cos\theta\sin\theta$ $= 25\sin 2\theta$ $\therefore P = AB + BC + CD$ $= 50\sin\theta + 50 + 25\sin 2\theta$ $= 25\sin 2\theta + 50\sin\theta + 50$ $(ii)\ \frac{dP}{d\theta} = 25\cos 2\theta(2) + 50\cos\theta$ $= 50\cos 2\theta + 50\cos\theta$ For stationary value, $\frac{dP}{d\theta} = 0$	M1 A1 M2 for Differen-Tiation M1 A1

$$50\cos 2\theta + 50\cos\theta = 0$$

$$\cos 2\theta + \cos\theta = 0$$ A1

$$2\cos^2\theta - 1 + \cos\theta = 0$$

$$2\cos^2\theta + \cos\theta - 1 = 0$$

$$(2\cos\theta - 1)(\cos\theta + 1) = 0$$

$$\cos\theta = \frac{1}{2} \quad or \quad \cos\theta = -1$$

$$\theta = \frac{\pi}{3}(1.0472)\ or\ \frac{5\pi}{3}(na) \quad or\ \theta = \pi(na)$$

When $\theta = \frac{\pi}{3}$ M1

$$P = 25\sin 2\left(\frac{\pi}{3}\right) + 50\sin\frac{\pi}{3} + 50$$ A1

$$= 114.95$$

$$= 115\text{ m}$$

(iii) $$\frac{d^2P}{d\theta^2} = 50(-\sin 2\theta)(2) + 50(-\sin\theta)$$

$$= -100\sin 2\theta - 50\sin\theta$$

When $\theta = \frac{\pi}{3}$ M1

$$\frac{d^2P}{d\theta^2} = -100\sin 2\left(\frac{\pi}{3}\right) - 50\sin\frac{\pi}{3}$$ A1

$$= -129.90 < 0$$

By 2nd derivative test, this value of P is maximum.

Alternatively,
Using 1st derivative test

θ	1.0	$\frac{\pi}{3} \approx 1.0472$	1.1
$\frac{dP}{d\theta}$	6.2077(+ve)	0	-6.7452(-ve)

By 1st derivative test, this value of P is maximum.