1 **(a)** The roots of the quadratic equation $2x^2 + x + 3 = 0$ are α and β. Find the quadratic equation whose roots are $\alpha^2 - 1$ and $\beta^2 - 1$. [5]

(b) Show that the equation $x^2 - (3-k)x + k = 4$ has real roots for all real values of k. [3]

(a)	Sum of roots $= \alpha + \beta = -\frac{1}{2}$ Product of roots $= \alpha\beta = \frac{3}{2}$ From $\alpha + \beta = -\frac{1}{2}$, $(\alpha+\beta)^2 = \left(-\frac{1}{2}\right)^2$ $\alpha^2 + 2\alpha\beta + \beta^2 = \frac{1}{4}$ $\alpha^2 + \beta^2 = \frac{1}{4} - 2\left(\frac{3}{2}\right)$ $= -2\frac{3}{4}$ Sum of new roots $= \alpha^2 - 1 + \beta^2 - 1$ $= \alpha^2 + \beta^2 - 2$ $= -2\frac{3}{4} - 2$ $= -4\frac{3}{4}$ Product of new roots $= (\alpha^2 - 1)(\beta^2 - 1)$ $= \alpha^2\beta^2 - \alpha^2 - \beta^2 + 1$ $= (\alpha\beta)^2 - (\alpha^2 + \beta^2) + 1$ $= \left(\frac{3}{2}\right)^2 - \left(-2\frac{3}{4}\right) + 1$ $= 6$ New quadratic equation: $x^2 - \left(-4\frac{3}{4}\right)x + 6 = 0$ $x^2 + 4\frac{3}{4}x + 6 = 0$ or $4x^2 + 19x + 24 = 0$	M1 A1 M1 M1 A1

(b)	Discriminant $= b^2 - 4ac$ $= [-(3-k)]^2 - 4(k-4)$ $= k^2 - 6k + 9 - 4k + 16$ $= k^2 - 10k + 25$ $= (k-5)^2$ Since $(k-5^2) \geq 0$, $x^2 - (3-k)x + k = 4$ has real roots for all real values of k.	M1 M1 A1

2 **(a)** Solve the equation $3\sqrt{2^x} + 12 = 3(2^{x-1})$. [4]

(b) Solve the equation $\log_3(8-x) + \log_3 x = 2\log_9 15$. [4]

(c) The mass, m grams, of a radioactive substance detected in a piece of stone is given by the formula $m = \beta e^{-kt}$, where β and k are constants, t is the time interval in months and $\beta \neq 0$.

(i) If the mass of the substance is reduced to half its original value four months after it was first being detected, find the value of k. [2]

(ii) Find the initial mass of the substance if its mass after one month is 0.25 g. [2]

(a)	$3\sqrt{2^x} + 12 = 3(2^{x-1})$ $3(2^x)^{\frac{1}{2}} + 12 = 3(2^x)\left(\frac{1}{2}\right)$ $2(2^x)^{\frac{1}{2}} + 8 = (2^x)$ Let $u = 2^x$. $2u^{\frac{1}{2}} + 8 = u$ $2u^{\frac{1}{2}} = u - 8$ $4u = u^2 - 16u + 64$ $u^2 - 20u + 64 = 0$ $(u-4)(u-16) = 0$ $u = 4$ or $u = 16$ $2^x = 2^2$ $2^x = 2^4$ $x = 2$ (NA) $x = 4$	M1 M1 A1 A1

(b)	$\log_3(8-x)+\log_3 x = 2\log_9 15$ $\log_3\left(8x-x^2\right)=\dfrac{2\log_3 15}{\log_3 9}$ $\log_3\left(8x-x^2\right)=\dfrac{2\log_3 15}{2\log_3 3}$ $8x-x^2=15$ $x^2-8x+15=0$ $(x-3)(x-5)=0$ $x=3$ or $x=5$	M2 – apply rules of logarithms M1 A1
(c) (i)	Initial mass of substance is when $t = 0$. $m=\beta e^{-k(0)}=\beta$ $0.5\beta=\beta e^{-k(4)}$ $0.5=e^{-k(4)}$ $\ln 0.5=-4k$ $k=\dfrac{\ln 2}{4}=0.173$	M1 A1
(c) (ii)	$0.25=\beta e^{-k}$ $0.25=\beta e^{-\frac{\ln 2}{4}}$ $\ln 0.25=\ln\beta+\ln e^{-\ln 2^{\frac{1}{4}}}$ $\ln\beta=\ln 0.25+\ln 2^{\frac{1}{4}}$ $\beta=0.25\times 2^{\frac{1}{4}}$ $=0.29730$ ≈ 0.297	M1 A1

3 **(a)** The first three terms in the binomial expansion $(1+kx)^n$ are $1+5x+\frac{45}{4}x^2+\ldots$.

Find the value of n and of k. [4]

(b) Find the term independent of x in the expansion of $x\left(2x-\frac{1}{2x^2}\right)^8$. [3]

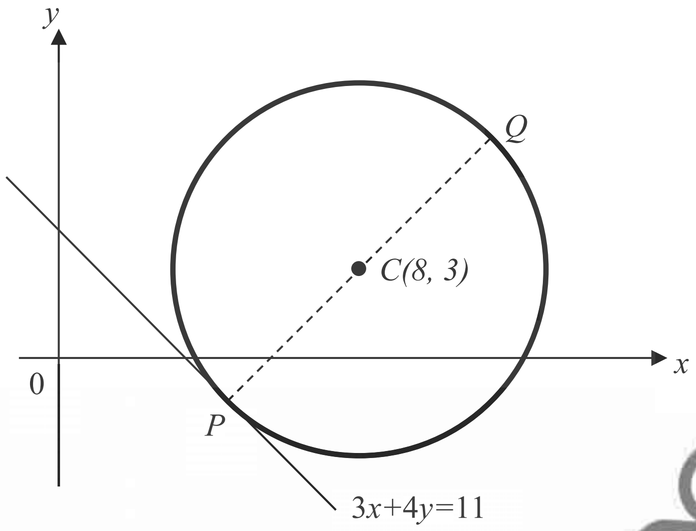
	$(1+kx)^n = 1+\binom{n}{1}(kx)^1+\binom{n}{2}(kx)^2+...$ $=1+nkx+\frac{n(n-1)(n-2)\times...}{2(n-2)(n-3)\times...}k^2x^2+...$ $=1+nkx+\frac{n(n-1)}{2}k^2x^2+...$ Comparing the coefficients of x and x^2, $nk=5$ $n=\frac{5}{k}$ ----------(1) $\frac{45}{4}=\frac{n(n-1)k^2}{2}$ $45=2n(n-1)k^2$ ----------(2) Subst. (1) into (2), $45=50-10k$ $k=\frac{1}{2}$ and $n=10$	M1 M1 M1 A1
(b)	Consider general term for $\left(2x-\frac{1}{2x^2}\right)^8$: $T_{r+1}=\binom{8}{r}(2x)^{8-r}\left(-\frac{1}{2x^2}\right)^r$ $=\binom{8}{r}(2)^{8-r}\left(-\frac{1}{2}\right)^r x^{8-3r}$ $x^{8-3r}=x^{-1}$ $8-3r=-1$ $9=3r$ $r=3$ Term independent of $x=\binom{8}{3}(2)^{8-3}\left(-\frac{1}{2}\right)^3$ $=-224$	 M1 M1 A1

4 The diagram shows a circle with centre $C(8, 3)$. PQ is a diameter of the circle and the equation of the tangent to the circle at P is given by $3x+4y=11$. Find

(i) the coordinates of P, [3]

(ii) the equation of the circle, and [2]

(iii) the coordinates of Q. [2]



(i)	Gradient of $PC = \frac{4}{3}$ Since P lies on $3x+4y=11 \Rightarrow y=-\frac{3}{4}x+\frac{11}{4}$, let the coordinates of $P\left(p,\ -\frac{3}{4}p+\frac{11}{4}\right)$. $\frac{-\frac{3}{4}p+\frac{11}{4}-3}{p-8}=\frac{4}{3}$ $-\frac{9}{4}p+\frac{33}{4}-9=4p-32$ $-6\frac{1}{4}p=-31\frac{1}{4}$ $p=5$ $y=-\frac{3}{4}(5)+\frac{11}{4}$ $=-1$ Coordinates of $P(5,\ -1)$	M1 M1 A1
(ii)	Radius of circle = $\sqrt{(8-5)^2+(3+1)^2}=5$ units Equation of circle: $(x-8)^2+(y-3)^2=25$	B1 B1
(iii)	Let the coordinates of Q be (a, b).	M1

	$\left(\frac{5+a}{2}, \frac{-1+b}{2}\right)=(8,3)$ $\frac{5+a}{2}=8$ $a=11$ $\frac{-1+b}{2}=3$ $b=7$ $\therefore Q(11,7)$	A1

5 Two variables, x and y, are related by an equation $y=k(x-1)^h$, where k and h are constants. The table below shows their experimental values obtained.

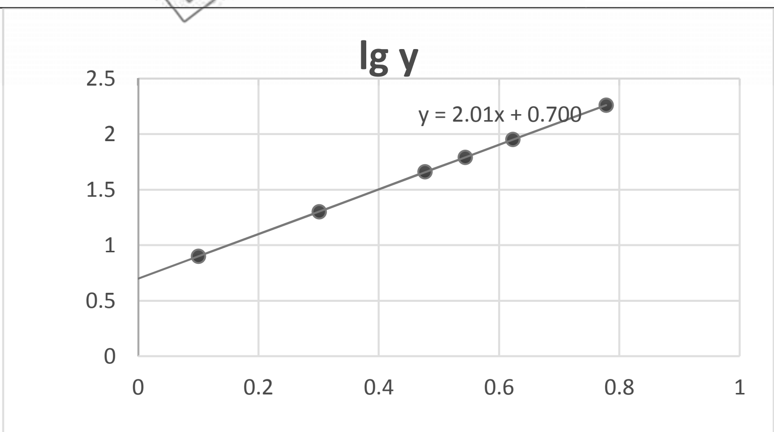
x	2.26	3.0	4.0	4.5	5.2	7.0
y	7.94	20.0	45.8	61.3	88.2	180

(i) Express the equation $y=k(x-1)^h$ in a form of $Y=mX+c$. [1]

(ii) Draw a straight line graph and use it to estimate the value of h and of k. [5]

(i)	$y=k(x-1)^h$ $\lg y=\lg\left[k(x-1)^h\right]$ $\lg y=h\lg(x-1)+\lg k$	B1
(ii)	Plot graph of lg y against lg (x–1).	B1

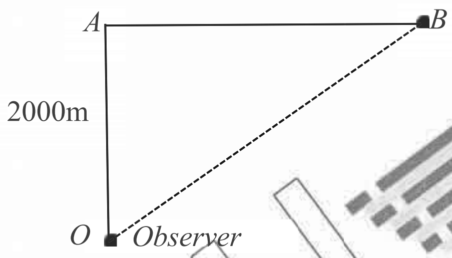
x	2.26	3.0	4.0	4.5	5.2	7.0
y	7.94	20.0	45.8	61.3	88.2	180
lg (x–1)	0.100	0.301	0.477	0.544	0.623	0.778
lg y	0.900	1.30	1.66	1.79	1.95	2.26



B1 – All points plotted correctly
B1 – Join all points with a best straight line and appropriate choice of scale

	$\lg k = 0.700$ $k = 10^{0.700}$ $k = 5.01 \pm 1.4$ $h = 2 \pm 0.2$	B1 B1

6 **(a)** An aeroplane is flying horizontally at an altitude of 2000 m and at a speed of 100 m/s. It passes directly above an observer on the ground. The diagram below shows the original position, A, of the aeroplane when it is directly above the observer and its position, B, t seconds later.



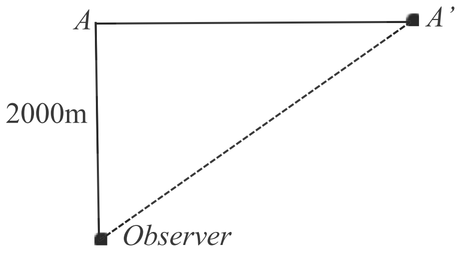
(i) Show that the distance, D m, between the aeroplane and the observer at time t is given by $D = 100\sqrt{400 + t^2}$. [2]

(ii) Hence, find how fast the distance, D, from the observer to the aeroplane is increasing 90 seconds later. [3]

(b) A solid cube has volume, V cm^3 and surface area, S cm^2,

(i) Show that $S = 6\sqrt[3]{V^2}$. [2]

(ii) The cube is heated and its volume is increasing at the rate of $0.008\,\text{cm}^3$/s, when its length is 3 cm. What is the rate of change of the surface area? [4]

(a)		

(i)	$D^2 = 2000^2 + (100t)^2$ $D^2 = 4000000 + 10000t^2$ Since $D > 0$, $\therefore D = \sqrt{10000(400 + t^2)}$ $D = 100\sqrt{400 + t^2}$ (shown)	M1 A1
(a) (ii)	$\frac{dD}{dt} = 100 \cdot \frac{1}{2} \cdot \frac{1(2t)}{\sqrt{400 + t^2}}$ $= \frac{100\text{t}}{\sqrt{400 + \text{t}^2}}$ When $t = 90s$, $\frac{dD}{dt} = \frac{100 \times 90}{\sqrt{400 + (90)^2}}$ ≈ 97.6 m/s	M1 A1 A1
(b) (i)	Let the length of the cube be x cm. $V = x^3$ ----(1) $S = 6x^2$ ----(2) From (1), $x = V^{1/3}$, substitute into (2) $S = 6\left(V^{1/3}\right)^2$ $= 6V^{\frac{2}{3}}$ $= 6\sqrt[3]{V^2}$	M1 A1
(a) (ii)	$S = 6\sqrt[3]{V^2}$ $\frac{dS}{dV} = 6 \times \frac{2}{3} V^{-\frac{1}{3}} = 4V^{-\frac{1}{3}}$ $V = x^3$ $= (3)^3$ $= 27$ cm^3 $\frac{dS}{dt} = \frac{dS}{dV} \times \frac{dV}{dt}$ $= 4(27)^{-\frac{1}{3}} \times 0.008$ $= 0.010667$ $= 0.0107$ cm^2/s	M1 A1 M1 A1

7 A cyclist is travelling along a straight road and passes a street light, L, with velocity v m/s, where $v = 5 + 3t - 2t^2$, and t, the time after passing the street light, is measured in seconds.

Find

(i) the maximum velocity of the cyclist within the first 3 seconds, [2]

(ii) the timing(s) when the cyclist is at instantaneous rest, [2]

(iii) the timing(s) when the cyclist is again at his initial speed, and [4]

(iv) the total distance travelled by the cyclist in the third second. [3]

(i)	$v = 5 + 3t - 2t^2$ $a = \frac{dv}{dt} = 3 - 4t$ At maximum velocity, $a = 0$ $3 - 4t = 0$ $t = \frac{3}{4}$ Maximum Velocity $= 5 + 3\left(\frac{3}{4}\right) - 2\left(\frac{3}{4}\right)^2$ $= \frac{49}{8}$ $= 6\frac{1}{8}$ m/s	M1 A1
(ii)	At instantaneous rest, $v = 0$ $5 + 3t - 2t^2 = 0$ $(5 - 2t)(t + 1) = 0$ $5 - 2t = 0$ $\quad t + 1 = 0$ $t = \frac{5}{2}$ $\quad t = -1$ (NA) The cyclist is at instantaneous rest at $t = \frac{5}{2}$ s.	M1 A1
(iii)	At $t = 0$, initial velocity $= 5$ m/s So speed is $\lvert v \rvert = 5$ $\lvert 5 + 3t - 2t^2 \rvert = 5$	

	$5+3t-2t^2=5$ $3t-2t^2=0$ $t(3-2t)=0$ $3-2t=0 \quad \text{or} \quad t=0$ $t=\frac{3}{2}$ $5+3t-2t^2=-5$ $10+3t-2t^2=0$ $t=\frac{-3\pm\sqrt{3^2-4(-2)(10)}}{2(-2)}$ $=-1.6085\ (\text{NA}) \text{ or } 3.1085$ The cyclist is at initial speed at $t=\frac{3}{2}$ s and $t=3.11$s.	M1 – solving the quadratic equation M1 – solving quadratic equation A1 – correct values A1
(iv)	$s=\int 5+3t-2t^2\,dt$ $=5t+\frac{3}{2}t^2-\frac{2}{3}t^3+c$ When $t=0, s=0$, so $c=0$ $s=5t+\frac{3}{2}t^2-\frac{2}{3}t^3$ $t=3$, $s=10.5$; $t=2.5$, $s=11.458$; $t=2$, $s=10.667$ $t=2,\ s=\frac{32}{3}=10.667$ $t=\frac{5}{2},\ s=\frac{275}{24}=11.458$ $t=3,\ s=\frac{21}{2}=10.5$ Distance travelled in the 3rd second $=\left(\frac{275}{24}-\frac{32}{3}\right)+\left(\frac{275}{24}-\frac{21}{2}\right)$ $=\frac{7}{4}$ m	M1 M1 A1
	OR	

	Distance travelled in the 3rd second	
	$= \int_{2}^{2.5} 5+3t-2t^2 dt + \left\| \int_{2.5}^{3} 5+3t-2t^2 dt \right\|$	
	$= \left[5t + \frac{3}{2}t^2 - \frac{2}{3}t^3 \right]_{2}^{2.5} + \left\| \left[5t + \frac{3}{2}t^2 - \frac{2}{3}t^3 \right]_{2.5}^{3} \right\|$	M1
	$= \left(5(2.5) + \frac{3}{2}(2.5)^2 - \frac{2}{3}(2.5)^3 \right) - \left(5(2) + \frac{3}{2}(2)^2 - \frac{2}{3}(2)^3 \right)$	
	$+ \left\| \left(5(3) + \frac{3}{2}(3)^2 - \frac{2}{3}(3)^3 \right) - \left(5(2.5) + \frac{3}{2}(2.5)^2 - \frac{2}{3}(2.5)^3 \right) \right\|$	M1
	$= \left(\frac{275}{24} - \frac{32}{3} \right) + \left(\frac{275}{24} - \frac{21}{2} \right)$	
	$= \frac{7}{4}$ m	A1

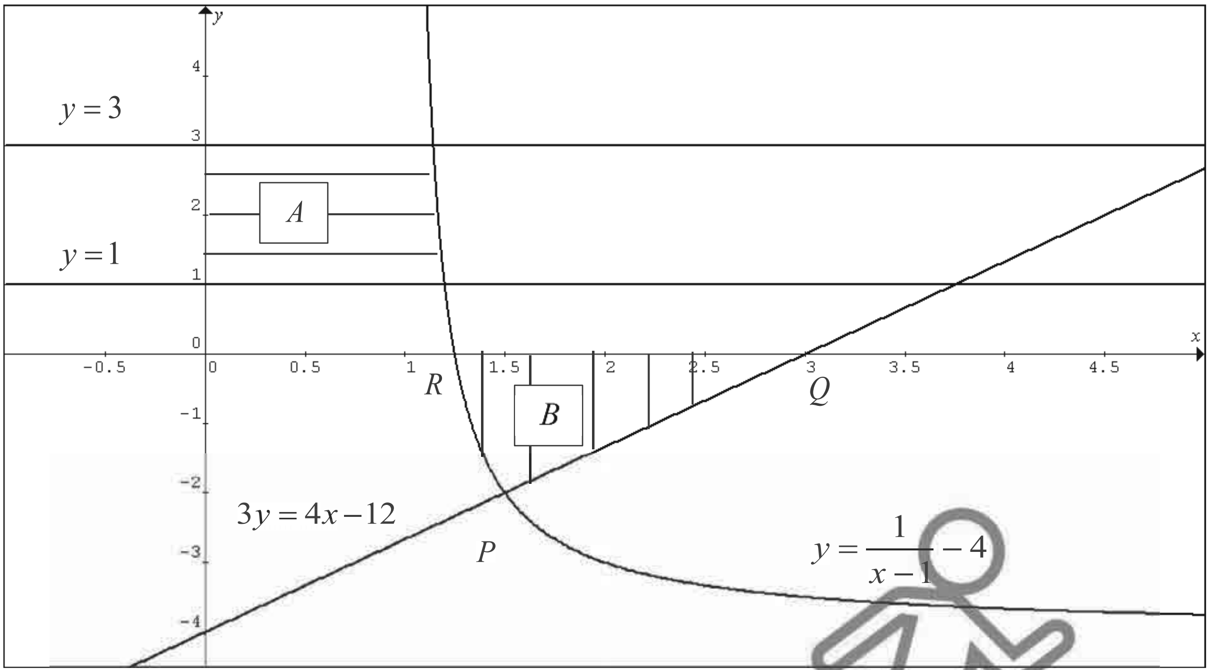
8 The sketch shows the graphs of the curve, $y = \frac{1}{x-1} - 4$, the lines $3y = 4x - 12$, $y = 1$, and $y = 3$. The curve and the line $3y = 4x - 12$ intersect at P. The curve cuts the $x-$axis at $R\left(\frac{5}{4}, 0\right)$. The $x-$intercept of the line $3y = 4x - 12$ is $Q\,(3, 0)$.

The region A is bounded by the curve, $y = \frac{1}{x-1} - 4$, the lines $y = 1$, $y = 3$, and the $y-$axis. The region B is bounded by the curve, the line $3y = 4x - 12$, and the $x-$axis.

Find

(i) the coordinates of P, and [3]

(ii) the area of A and of B. [7]



(i)	$y=\dfrac{1}{x-1}-4$ ------- (1) $\quad 3y=4x-12$ ----------- (2)	
	Substitute (1) into (2)	
	$3\left(\dfrac{1}{x-1}-4\right)=4x-12$	
	$\dfrac{3}{x-1}-12=4x-12$	
	$\dfrac{3}{x-1}=4x$	M1
	$4x^2-4x-3=0$	
	$(2x-3)(2x+1)=0$	
	$2x-3=0 \qquad 2x+1=0$	
	$x=\dfrac{3}{2}$ or $x=-\dfrac{1}{2}$	M1
	$x=\dfrac{3}{2},\ 3y=4\left(\dfrac{3}{2}\right)-12,\quad y=-2 \quad P\left(\dfrac{3}{2},-2\right)$	A1

(ii)	$y=\frac{1}{x-1}-4$ $y+4=\frac{1}{x-1}$ $x-1=\frac{1}{y+4}$ $x=\frac{1}{y+4}+1$	M1
	$A=\int_1^3\left(\frac{1}{y+4}+1\right)dy$	M1
	$=\left[\ln(y+4)+y\right]_1^3$ $=(\ln(3+4)+3)-(\ln(1+4)+1)$	M1
	$=\ln 7-\ln 5+2$ $=2.3365$ $=2.34\text{ units}^2$	A1
(ii)	$B=\left\|\int_{5/4}^{3/2}\left(\frac{1}{x-1}-4\right)dx\right\|+\frac{1}{2}\times\left(3-\frac{3}{2}\right)\times 2$	M1
	$=\left\|\left[\ln(x-1)-4x\right]_{5/4}^{3/2}\right\|+\frac{3}{2}$ $=\left\|\left(\ln\left(\frac{3}{2}-1\right)-4\left(\frac{3}{2}\right)\right)-\left(\ln\left(\frac{5}{4}-1\right)-4\left(\frac{5}{4}\right)\right)\right\|+\frac{3}{2}$	M1
	$=0.30685+\frac{3}{2}$ $=1.80685$ $=1.81\text{ units}^2$	A1

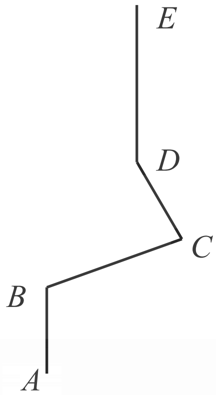
9 The sketch shows the journey of a kayak. The kayak heads due north from a point A for 200 m, to reach B and heads at a bearing of θ for 400 m to reach C. It then makes a $90°$ turn and travels for 300 m to D, after which it heads due north again for 500 m, to end at E. The total distance of the kayak due north from A is L m.

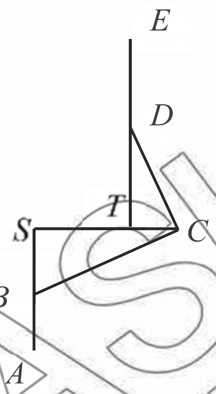
(i) Show that $L=700+400\cos\theta+300\sin\theta$ [3]

(ii) Express L in the form $k+R\cos(\theta-\alpha)$ where k and R are positive constants, and $0°<\alpha<90°$. [3]

(iii) Determine the value of θ if the kayak ends at 1.15 km due north of A. [2]

(iv) If the kayak travelled for 45 minutes, what could be the maximum average speed heading due north, and the corresponding value of θ? [3]



(i)	 $\angle CSB = \angle DTC = \angle BCD = 90°$ $\angle SBC = \angle TCD = \theta$ $L = AB + BS + TD + DE$ $= 200 + 400\cos\theta + 300\sin\theta + 500$ $= 700 + 400\cos\theta + 300\sin\theta$	M1 M1 A1
(ii)	$400\cos\theta + 300\sin\theta = R\cos(\theta - \alpha)$ $= R\cos\theta\cos\alpha + R\sin\theta\sin\alpha$ $R\cos\alpha = 400 \quad R\sin\theta = 300$ $R^2 = 400^2 + 300^2$ $R = 500$ $\tan\alpha = \dfrac{300}{400}$ $\alpha = 36.870°$ $L = 700 + 400\cos\theta + 300\sin\theta$ $= 700 + 500\cos(\theta - 36.870°)$	M1 A1 – R and α A1

(iii)	$700 + 500\cos(\theta - 36.870°) = 1150$ $500\cos(\theta - 36.870°) = 450$ $\cos(\theta - 36.870°) = \frac{450}{500}$ $\theta - 36.870° = 25.842°, -25.842°$ $\theta = 62.711°, 11.028°$ $= 062.7°, 011.0°$	M1 A1
(iv)	Maximum speed occurs for maximum L. Maximum $L = 700 + 500 = 1200$ m. $\text{Maximum Speed} = \frac{1200}{0.75}$ $= 1600$ $= 1.6$ km/h $\cos(\theta - 36.870°) = 1$ $(\theta - 36.870°) = 0°$ $\theta = 36.870°$ $= 036.9°$	M1 A1 A1

10 The equation of a curve is given by $y = (2x-9)\sqrt{x^2+1}$.

(i) Express $\frac{dy}{dx}$ in the form $\frac{ax^2+bx+c}{\sqrt{x^2+1}}$ where a, b and c are real constants. [4]

(ii) Find the range of values of x for which y is a decreasing function of x. [3]

(iii) Determine the minimum point of the curve. [3]

(i)	$y = (2x-9)\sqrt{x^2+1}$ $\frac{dy}{dx} = \sqrt{x^2+1}\,(2) + (2x-9)\frac{1}{2}(x^2+1)^{-1/2}(2x)$ $= \sqrt{x^2+1}\,(2) + \frac{(2x-9)x}{\sqrt{x^2+1}}$ $= \frac{2x^2+2+2x^2-9x}{\sqrt{x^2+1}}$ $= \frac{4x^2-9x+2}{\sqrt{x^2+1}}$	M2 M1 A1
(ii)	For a decreasing function, $\frac{dy}{dx} < 0$	

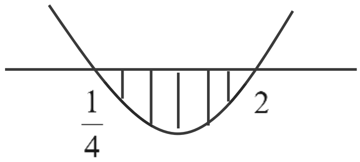
$\frac{4x^2-9x+2}{\sqrt{x^2+1}}<0$

Since $\sqrt{x^2+1}>0$

$4x^2-9x+2<0$ M1

$(4x-1)(x-2)<0$ M1

$\frac{1}{4}<x<2$ A1



(iii)

$\frac{dy}{dx}=0$

$\frac{4x^2-9x+2}{\sqrt{x^2+1}}=0$

$\frac{(4x-1)(x-2)}{\sqrt{x^2+1}}=0$

$x=\frac{1}{4}$ or $x=2$

x	$\frac{1}{4}^-$	$\frac{1}{4}$	$\frac{1}{4}^+$		x	2^-	2	2^+
$\frac{dy}{dx}$	+	0	−		$\frac{dy}{dx}$	−	0	+
	/	—	\			\	—	/

M1

$x=\frac{1}{4}$ gives a maximum point. $x=2$ gives a minimum point.

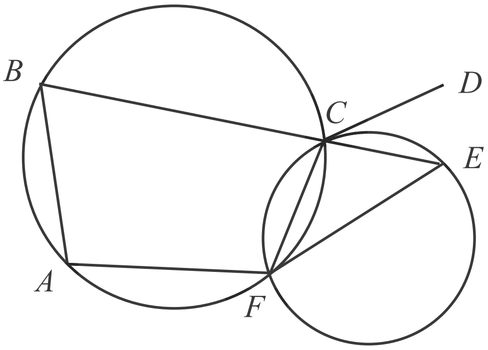
$y=(2(2)-9)\sqrt{(2)^2+1}$ M1

$=-5\sqrt{5}$

Minimum point is $(2,-5\sqrt{5})$ A1

[Turnover

11 The diagram shows two circles that intersect each other at points C and F. The points A and B lie on the circumference of the larger circle. The point E lies on the circumference of the smaller circle such that BCE is a straight line. Line CD is a tangent to the smaller circle at D. The lines CE and CF are of equal length.



(i) Prove that lines CD and FE are parallel. [3]

(ii) Show that $\angle BAF + 2\angle DCE = 180°$. [4]

(i)	$\angle DCE = \angle CFE$ (Tangent Chord Theorem / Alternate Segment Theorem. Since $CE = CF$, triangle CFE is an isosceles triangle. And $\angle FEC = \angle CFE$ (Base angles of isosceles triangle) So $\angle DCE = \angle FEC$, they are alternate angles, Hence lines CD and FE are parallel.	M1 M1 A1
(ii)	$\angle FCB = \angle CFE + \angle FEC$ (Exterior Angle = Sum of Interior Opposite Angles) $\angle CFE = \angle FEC$ $= \angle DCE$ $\angle FCB = \angle DCE + \angle DCE$ $= 2\angle DCE$ $\angle BAF + \angle FCB = 180°$ (Angles in opposite segments) Therefore $\angle BAF + 2\angle DCE = 180°$	M1 M1 M1 A1

End of Paper