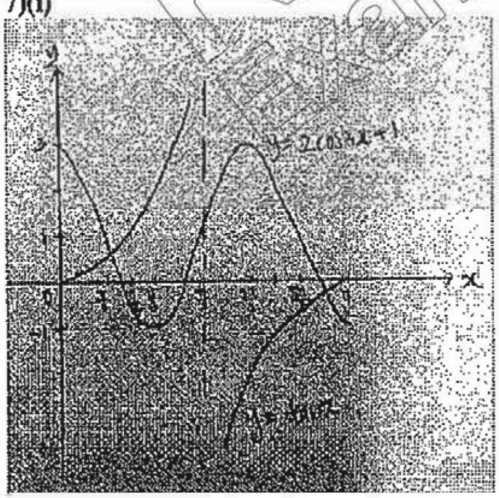
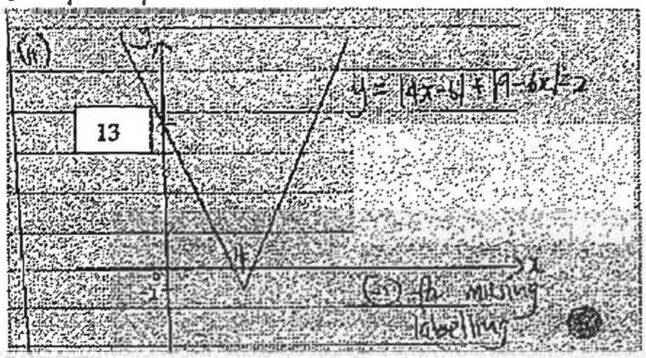
Marking Scheme

Answer	Marks	Remarks
1(i) $\tan A = -\frac{4}{3}$	A1	
(ii) $\sin(-B)$ $= -\sin B$ $= \frac{5}{13}$	A1	
(iii) $\operatorname{cosec}(180^\circ - B)$ $= \frac{1}{\sin B}$	M1	Simplifying to $\frac{1}{\sin B}$
$= -\frac{13}{5}$	A1	
2) $6\sin x + 4\tan x = 2\sec x + 3$ $6\sin x + 4\frac{\sin x}{\cos x} = 2\frac{1}{\cos x} + 3$	M1	Converting to sin and cos
$6\sin x\cos x + 4\sin x - 2 - 3\cos x = 0$ $2\sin x(3\cos x + 2) - (3\cos x + 2) = 0$ $(3\cos x + 2)(2\sin x - 1) = 0$	M1	Factorizing by grouping
$\cos x = -\frac{2}{3}$ *or* $\sin x = \frac{1}{2}$ $ref\angle = 48.189^\circ$ $ref\angle = 30^\circ$ $x = -131.8^\circ, 131.8^\circ$ $x = 30^\circ, 150^\circ$	A2	A1 for each set of answer
3) $y^2 = 72x$ ---(1) $y = 3x^2$ --- (2) Sub (2) into (1) $(3x^2)^2 = 72x$	M1	Correct substitution method
$9x^4 - 72x = 0$ $9x(x^3 - 8) = 0$ $x = 0$ *or* $x = 2$ $y = 0$ *or* $y = 12$	M1 M1	Solving for x values Solving for y values
$Grad = \frac{12-0}{2-0}$	M1	Finding the gradient using x, y val found
$= 6$ $\therefore y = 6x$	A1	
4) $m - 3 > 0$ *and* $3^2 - 4(m-3)(m+1) < 0$ $m > 3$ $9 - 4(m^2 - 2m - 3) < 0$	M2	M1: Coefficient of $x^2 > 0$ M1: Discriminant < 0
$4m^2 - 8m - 21 > 0$ $(2m+3)(2m-7) > 0$	M1	Factorizing to solve
$m < -1\frac{1}{2}$ *or* $m > 3\frac{1}{2}$	M1	Solution to quadratic inequality
$\therefore m > 3\frac{1}{2}$	A1	

5) $\frac{4x^3-2x^2-13x+13}{2x^2-2}=2x-1+\frac{-9x+11}{2(x-1)(x+1)}$ $\frac{-9x+11}{2(x-1)(x+1)}=\frac{A}{2(x-1)}+\frac{B}{x+1}$ $-9x+11=A(x+1)+2B(x-1)$ $x=1,\quad 2=2A$ $A=1$ $x=-1,\quad 20=-4B$ $B=-5$ $\frac{4x^3-2x^2-13x+13}{2x^2-2}=2x-1+\frac{1}{2(x-1)}-\frac{5}{x+1}$	M1 M1 M2 A1	**Simplifying complex fraction using long division/comparing coefficients** **Use correct partial fraction forms** accept $\frac{-9x+11}{2(x-1)(x+1)}=\frac{A}{(x-1)}+\frac{B}{2(x+1)}$ **Solving for A and B using substitution of values or comparing coefficients, 1 mark each**
6) $6\times\frac{1}{2}(2+\sqrt{3})^2\sin 60°\times height=\frac{3}{2}(17\sqrt{3}+30)$ $3(7+4\sqrt{3})\times\frac{\sqrt{3}}{2}\times height=\frac{3}{2}(17\sqrt{3}+30)$ $Height=\frac{17\sqrt{3}+30}{\sqrt{3}(7+4\sqrt{3})}$ $=\frac{17\sqrt{3}+30}{12+7\sqrt{3}}$ $=\frac{17\sqrt{3}+30}{12+7\sqrt{3}}\times\frac{12-7\sqrt{3}}{12-7\sqrt{3}}$ $=\frac{357-204\sqrt{3}+210\sqrt{3}-360}{3}$ $=\frac{6\sqrt{3}-3}{3}$ $=(2\sqrt{3}-1)cm$	M2 M1 M1 M1 A1	**M1:finding angle as 60** **M1: forming equation using volume** **Evaluating exact value of sin60** **Making height the subject** **Rationalizing using conjugate surds**
7)(i)  **(ii)(a) 2** **(ii)(b) 1**	G4 B1 B1	**For cosine graph:** **G1: correct shape and no of cycles** **G1: correct max/min/axis values** **For tangent graph:** **G1: correct shape and no of cycles** **G1: correct asymptotes and passing through** $\left(\frac{\pi}{4},1\right)$ **and** $\left(\frac{3\pi}{4},-1\right)$ **Minus 1 mark for any missing labeling** **Not awarded if graphs are wrong.**

8(i) $\lvert 4x-6\rvert+\lvert 9-6x\rvert$ $=\lvert 2(2x-3)\rvert+\lvert -3(2x-3)\rvert$ $=2\lvert 2x-3\rvert+3\lvert 2x-3\rvert$ $=5\lvert 2x-3\rvert$	M1 A1	Factoring out constant for each modulus portion to form 2x − 3 Show relevant workings to reduce to $5\lvert 2x-3\rvert$
(ii) $y=\lvert 4x-6\rvert+\lvert 9-6x\rvert-2$ $y=5\lvert 2x-3\rvert-2$ 	M1 G2	Rewriting equation of y G1: V shape graph with vertex below x-axis G1: correct values for y intercept and vertex. Minus 1 mark for any missing labeling
(iii) Grad of line joining (0, −5) to vertex $=\dfrac{-5-(-2)}{0-1\frac{1}{2}}$ $=2$ $\therefore 2<m<10$	M1 A1	Finding lower bound for m using line joining (0, −5) to vertex
9(i) $y=\ln\sqrt{\left(\dfrac{x^2+6x}{3x^2+16x-12}\right)^3}$ $=\dfrac{3}{2}\ln\dfrac{x^2+6x}{3x^2+16x-12}$ $=\dfrac{3}{2}\ln\dfrac{x(x+6)}{(x+6)(3x-2)}$ $=\dfrac{3}{2}[\ln x-\ln(3x-2)]$ $\dfrac{dy}{dx}=\dfrac{3}{2}\left[\dfrac{1}{x}-\dfrac{3}{3x-2}\right]$ $=\dfrac{3}{2}\left(\dfrac{-2}{x(3x-2)}\right)$ $=-\dfrac{3}{x(3x-2)}$	M1 M1 M1 M1 A1	Writing index as coefficient Factorising to reduce complex fraction Rewriting as a difference Correct differentiation of ln functions
(ii) For $x>\dfrac{3}{2}$, $3x-2>0$ $x(3x-2)>0$ $\dfrac{dy}{dx}=-\dfrac{3}{x(3x-2)}<0$ $\therefore \dfrac{dy}{dx}\neq 0$ Since $\dfrac{dy}{dx}\neq 0$, the curve has no turning points	M1 A1	Reasoning for $\dfrac{dy}{dx}=-\dfrac{3}{x(3x-2)}<0$ Or any reasonable explanation. Do not accept if attempt to solve for $\dfrac{dy}{dx}=0$ and claims no solution. Stating $\dfrac{dy}{dx}\neq 0$ with conclusion

Solution	Marks	Remarks
10(a) $\frac{d}{dx}\left[\cot^2\left(\frac{\pi}{2}-2x\right)\sin 2x\right]$		
$=\frac{d}{dx}\left[\tan^2 2x \sin 2x\right]$	M1	Using $\left[\cot\left(\frac{\pi}{2}-2x\right)=\tan 2x\right]$ or use of complimentary angles relationship
$=\frac{d}{dx}\left[\frac{\sin^3 2x}{\cos^2 2x}\right]$	M1	**Converting to sine and cosine**
$=\frac{\cos^2 2x(3\sin^2 2x)(2\cos 2x)-\sin^3 2x(2\cos 2x)(-2\sin 2x)}{\cos^4 2x}$	M1	**Correct application of quotient/product rule and differentiation of trigo**
$=\frac{2\cos 2x\sin^2 2x(3\cos^2 2x+2\sin^2 2x)}{\cos^4 2x}$		
$=\frac{2\sin^2 2x(3\cos^2 2x+2(1-\cos^2 2x))}{\cos^3 2x}$ $\frac{2\sin^2 2x(2+\cos^2 2x)}{\cos^3 2x}$	A1	**Use of $\sin^2 2x+\cos^2 2x=1$ or relevant identities to reduce to required answer**
(b) $\int_0^{\frac{\pi}{3}}(5\tan^2 2x+3)\,dx$		
$=\int_0^{\frac{\pi}{3}}(5\sec^2 2x-2)\,dx$	M1	**Use of identity to reduce to expression in sec**
$=\left[\frac{5}{2}\tan 2x-2x\right]_0^{\frac{\pi}{3}}$	M1	**Correct integration**
$=-\frac{5\sqrt{3}}{2}-\frac{2\pi}{3}$	A1	
11) $\frac{dy}{dx}=4x^3-9x^2$	M1	**Correct differentiation**
At stationary pt, $\frac{dy}{dx}=0$ $4x^3-9x^2=0$	M1	**Setting first derivative to zero**
$x^2(4x-9)=0$ $x=0$ *or* $x=2\frac{1}{4}$	M1	**Solving for x values**
$y=1$ *or* $y=-7\frac{139}{256}$		
Coordinates are $(0,1)$ and $\left(2\frac{1}{4},-7\frac{139}{256}\right)$	A2	**1 mark each for coordinates, Accept (2.25, -7.54)**

At $(0,1)$,

x	-0.1	0	0.1
dy/dx	<0	0	<0
slope	\	-	\

$(0,1)$ is a point of inflexion

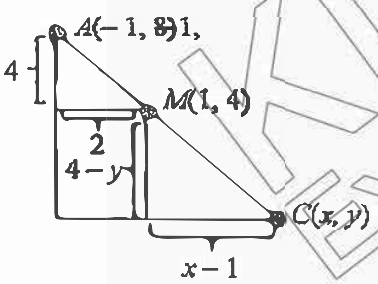
M1A1 — **Use of first derivative test to conclude point of inflexion**

At $\left(2\frac{1}{4},-7\frac{139}{256}\right)$

x	2.24	2.25	2.26
dy/dx	<0	0	>0
slope	\	-	/

$\left(2\frac{1}{4},-7\frac{139}{256}\right)$ is a minimum point.

A1 — **Use of appropriate test to conclude min point**

12(i) Midpoint $BD = (1, 4)$ Gradient $BD = \frac{1}{2}$	M1	**Finding midpoint of BD**
Gradient $AC = -2$ Sub $(1, 4)$ into $y = -2x + c$ $c = 6$	M1	**Finding gradient of AC using gradient of perpendicular lines concept**
Equation of AC is $y = -2x + 6$ At A, $x = -1$, $y = 8$	M1	**Finding equation of AC**
Coordinates of $A = (-1, 8)$	A1	
OR Midpoint $BD = (1, 4)$ Gradient $BD = \frac{1}{2}$	M1	**Finding midpoint of BD**
Gradient $AC = -2$ Let coordinate of $A = (-1, a)$	M1	**Finding gradient of AC using gradient of perpendicular lines concept**
$\frac{a-4}{-1-1} = -2$ $a = 8$	M1	**Form equation using gradient**
Coordinates of $A = (-1, 8)$	A1	
OR Let coordinate of $A = (-1, a)$	M1	**Identify y- coordinate as -1**
$\sqrt{(-1-(-1))^2 + (a-3)^2} = \sqrt{(3-(-1))^2 + (a-5)^2}$ $a^2 - 6a + 9 = 16 + a^2 - 10a + 25$	M1	**Use of distance formula to equate AB to AD**
$4a = 32$ $a = 8$	M1	**Solving for a**
Coordinates of $A = (-1, 8)$	A1	
(ii) Let M be the midpoint of BD. $AM : MC = 2 : 3$	M1	**Derive ratio of base using ratio of area and common height**
 		
Using similar triangles, $\frac{x-1}{2} = \frac{3}{2}$ $x = 4$	M1	**Using similar triangle ratio to find x value**
$\frac{4-y}{4} = \frac{3}{2}$ $y = -2$	M1	**Using similar triangle ratio to find y value**
Coordinates of $C = (4, -2)$	A1	

OR Let coordinates of $C=(x,y)$		
$3\times\frac{1}{2}\begin{vmatrix}-1 & 3 & -1 & -1\\ 3 & 5 & 8 & 3\end{vmatrix}=2\times\frac{1}{2}\begin{vmatrix}-1 & x & 3 & -1\\ 3 & y & 5 & 3\end{vmatrix}$	M1	Finding area and forming equation
$30=-4y+2x+14$ $x-2y=8----(1)$	M1	Reduce to linear equation
$y=-2x+6----(2)$ $Sub\,(2)$ into (1)		
$x=-4,\quad y=-2$ Coordinates of C $=(4,-2)$	M1 A1	Solving simultaneous equation
OR *if midpoint and equation of AC not found in part (i)		
Midpoint $BD=(1,4)$	M1	Finding Midpoint
Gradient $BD=\frac{1}{2}$ Gradient $AC=-2$ Sub $(1,4)$ into $y=-2x+c$ $c=6$		
Equation of AC is $y=-2x+6$	M1	Find equation of AC
Let coordinates of $C=(c,-2c+6)$ Ratio of height $=2:3$		
$\frac{\sqrt{(1-(-1))^2+(4-8)^2}}{\sqrt{(c-1)^2+(-2c+6-4)^2}}=\frac{2}{3}$	M1	Using height ratio
$c^2-2c-8=0$ $(c-4)(c+2)=0$		
$c=4\quad or\quad c=-2\,(NA)$ Coordinates of C $=(4,-2)$	A1	

13(i) $\angle ATD = \angle BTA$ (common angle) $\angle DAT = \angle ABD$ (alt seg thm) $= \angle ABT$ (common angle) $\angle ADT = \angle BAT$ (angle sum of triangle)	M2	Minus 1 mark for each missing set of equal corresponding angles
Since there are 3 pairs of corresponding angles which are equal, triangle ATD is similar to triangle BTA (proved)	A1	Appropriate conclusion with reasons
(ii) $\frac{AT}{BT} = \frac{DT}{AT}$ (corr. sides of similar triangles) $AT^2 = BT \times DT$ $= TB \times DT \quad (\because BT = TB)$	M1	Using similar triangle ratio to derive $AT^2 = TB \times DT$
Since $BD : BT = 2 : 5$, $DT : BD = 3 : 2$ $DT = \frac{3}{2} BD$	M1	Use of $BD : BT = 2 : 5$ to derive $DT = \frac{3}{2} BD$
Since $BE : ED = 3 : 2$, $BE : BD = 3 : 5$ $BD = \frac{5}{3} BE$	M1	Use of $BE : ED = 3 : 2$ to derive $BD = \frac{5}{3} BE$
$DT = \frac{3}{2}\left(\frac{5}{3} BE\right)$ $= \frac{5}{2} BE$	M1	Express DT in terms of BE
$AT^2 = TB \times \frac{5}{2} BE$ $2AT^2 = 5(TB \times BE)$ (*proved*)	A1	Show relevant workings to derive final answer