Name ________________________________ ( )	Class 4 _______

ANGLICAN HIGH SCHOOL
PRELIMINARY EXAMINATION 2017
SECONDARY FOUR

S4

ADDITIONAL MATHEMATICS 4047/02
Paper 2

FRIDAY 21 JULY 2017
2 hours 30 minutes

Additional Materials: Writing Paper × 07 Graph Paper × 01

READ THESE INSTRUCTIONS FIRST

Write your name and index number on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

At the end of the examination, attach this cover page on top of your answer scripts.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is **100**.

For Examiner's Use

Question	1	2	3	4	5	6	7	8	9	10	11
Marks											

Table of Penalties		Qn. No.		
Presentation	–1			
Units	–1			
Significant Figures	–1		Parent's/ Guardian's Name/ Signature/ Date	100

This question paper consists of 8 printed pages.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the quadratic equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} .$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n ,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \tfrac{1}{2} ab \sin C$$

1 **(a)** The roots of the quadratic equation $2x^2 + x + 3 = 0$ are α and β. Find the quadratic equation whose roots are $\alpha^2 - 1$ and $\beta^2 - 1$. [5]

(b) Show that the equation $x^2 - (3-k)x + k = 4$ has real roots for all real values of k. [3]

2 **(a)** Solve the equation $3\sqrt{2^x} + 12 = 3\left(2^{x-1}\right)$. [4]

(b) Solve the equation $\log_3(8-x) + \log_3 x = 2\log_9 15$. [4]

(c) The mass, m grams, of a radioactive substance detected in a piece of stone is given by the formula $m = \beta e^{-kt}$, where β and k are constants, t is the time interval in months and $\beta \neq 0$.

(i) If the mass of the substance is reduced to half its original value four months after it was first being detected, find the value of k. [2]

(ii) Find the initial mass of the substance if its mass after one month is 0.25 g. [2]

3 **(a)** The first three terms in the binomial expansion $(1+kx)^n$ are $1 + 5x + \frac{45}{4}x^2 + \ldots$. Find the value of n and of k. [4]

(b) Find the term independent of x in the expansion of $x\left(2x - \frac{1}{2x^2}\right)^8$. [3]

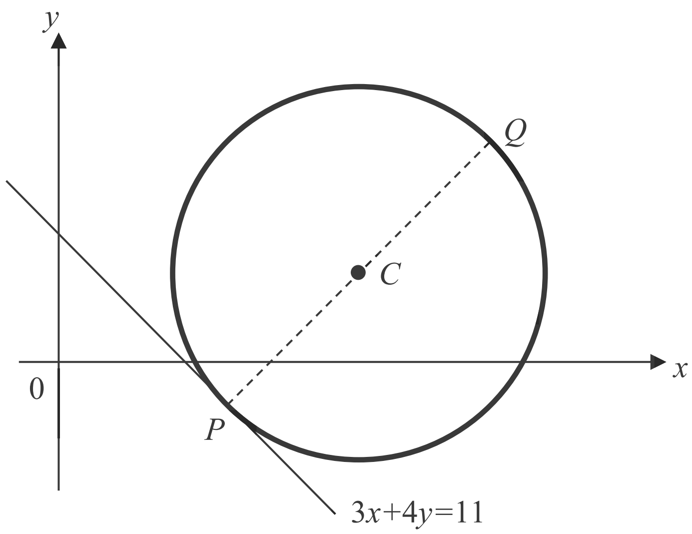
[Turnover

4 The diagram shows a circle with centre $C(8, 3)$. PQ is a diameter of the circle and the equation of the tangent to the circle at P is given by $3x+4y=11$. Find

(i) the coordinates of P, [3]

(ii) the equation of the circle, and [2]

(iii) the coordinates of Q. [2]



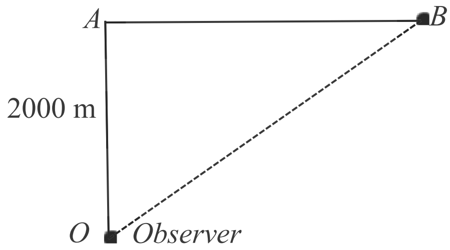
5 Two variables, x and y, are related by an equation $y=k(x-1)^h$, where k and h are constants. The table below shows their experimental values obtained.

x	2.26	3.0	4.0	4.5	5.2	7.0
y	7.94	20.0	45.8	61.3	88.2	180

(i) Express the equation $y=k(x-1)^h$ in a form of $Y=mX+c$. [1]

(ii) Draw a straight line graph and use it to estimate the value of h and of k. [5]

6 **(a)** An aeroplane is flying horizontally at an altitude of 2000 m and at a speed of 100 m/s. It passes directly above an observer, O, on the ground. The diagram below shows the original position, A, of the aeroplane when it is directly above the observer and its position, B, t seconds later.



(i) Show that the distance, D m, between the aeroplane and the observer at time t is given by $D = 100\sqrt{400 + t^2}$. [2]

(ii) Hence, find how fast the distance, D, from the observer to the aeroplane is increasing 90 seconds later. [3]

(b) A solid cube has volume, V cm^3 and surface area, S cm^2.,

(i) Show that $S = 6\sqrt[3]{V^2}$. [2]

(ii) The cube is heated and its volume is increasing at the rate of 0.008 cm^3/s, when its length is 3 cm. What is the rate of change of the surface area? [4]

7 A cyclist travels along a straight road and passes a street light, L, with velocity v m/s, where $v = 5 + 3t - 2t^2$, and t, the time after passing the street light, is measured in seconds.
Find

(i) the maximum velocity of the cyclist within the first 3 seconds, [2]

(ii) the timing(s) when the cyclist is at instantaneous rest, [2]

(iii) the timing(s) when the cyclist is again at his initial speed, and [4]

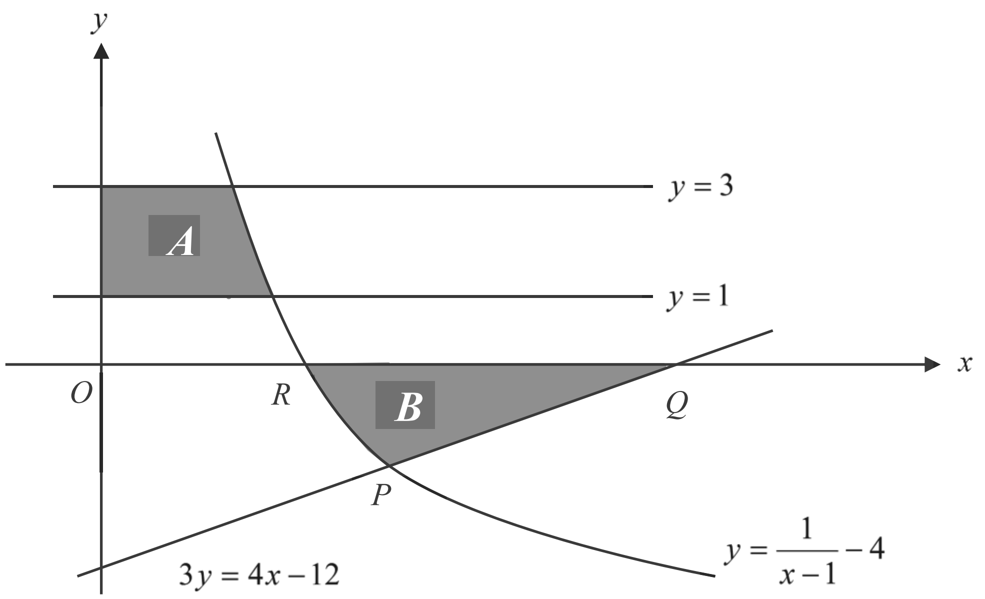
(iv) the total distance travelled by the cyclist in the third second. [3]

[Turnover

8 The sketch shows the graphs of the curve, $y = \frac{1}{x-1} - 4$, the lines $3y = 4x - 12$, $y = 1$, and $y = 3$. The curve and the line $3y = 4x - 12$ intersect at P. The curve cuts the $x-$axis at $R\left(\frac{5}{4}, 0\right)$. The $x-$intercept of the line $3y = 4x - 12$ is $Q(3, 0)$. The region A is bounded by the curve, $y = \frac{1}{x-1} - 4$, the lines $y = 1$, $y = 3$, and the $y-$axis. The region B is bounded by the curve, the line $3y = 4x - 12$, and the $x-$axis. Find

(i) the coordinates of P, and [3]

(ii) the area of A and of B. [7]



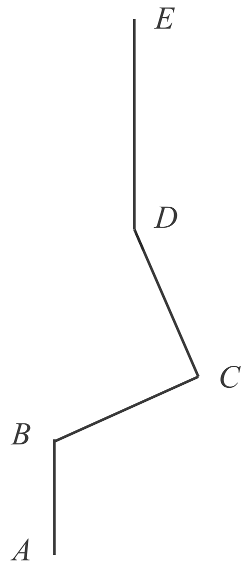
9 The sketch shows the journey of a kayak. The kayak heads due north from a point A for 200 m, to reach B and heads at a bearing of θ for 400 m to reach C. It then makes a $90°$ turn and travels for 300 m to D, after which it heads due north again for 500 m, to end at E. The total distance of the kayak due north from A is L m.

(i) Show that $L = 700 + 400\cos\theta + 300\sin\theta$. [3]

(ii) Express L in the form $k + R\cos(\theta - \alpha)$ where k and R are positive constants, and $0° < \alpha < 90°$. [3]

(iii) Determine the value of θ if the kayak ends at 1.15 km due north of A. [2]

(iv) If the kayak travelled for 45 minutes, what could be the maximum average speed heading due north, and the corresponding value of θ? [3]



10 The equation of a curve is given by $y = (2x-9)\sqrt{x^2+1}$.

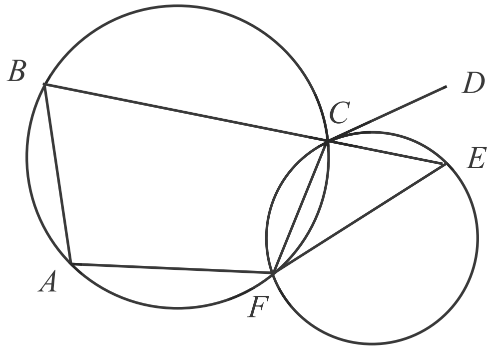
(i) Express $\dfrac{dy}{dx}$ in the form $\dfrac{ax^2+bx+c}{\sqrt{x^2+1}}$ where a, b and c are real constants. [4]

(ii) Find the range of values of x for which y is a decreasing function of x. [3]

(iii) Determine the minimum point of the curve. [3]

[Turnover

11 The diagram shows two circles that intersect each other at points C and F. The points A and B lie on the circumference of the larger circle. The point E lies on the circumference of the smaller circle such that BCE is a straight line. Line CD is a tangent to the smaller circle at C. The lines CE and CF are of equal length.



(i) Prove that lines CD and FE are parallel. [3]

(ii) Show that $\angle BAF + 2\angle DCE = 180°$. [4]

End of Paper