Qns	Solutions	Marks Allocation	Remarks
1a	$y=\frac{e^{2x}}{2x-3}$ $\frac{dy}{dx}=\frac{(2x-3)(2e^{2x})-e^{2x}(2)}{(2x-3)^2}$ $=\frac{2e^{2x}[(2x-3)-1]}{(2x-3)^2}$ $=\frac{e^{2x}(4x-8)}{(2x-3)^2}$ or $=\frac{4e^{2x}(x-2)}{(2x-3)^2}$ Since $e^{2x}>0$ and $(2x-3)^2>0$ for all $x>2$ When $x>2,\ x-2>0$ $\therefore \frac{dy}{dx}>0$ for all $x>2$. $\therefore y$ is in increasing function for $x>2$.	M1 M1 M1 A1	Chain Rule Quotient Rule Conclude $e^{2x}>0$ and $(2x-3)^2>0$ for all x and for $x>2$, $4x-8>0$ $\therefore \frac{dy}{dx}>0$ for all $x>2$. $\therefore y$ is in increasing function for $x>2$.
1b	$y=(2x+5)(x-4)^2$ $\frac{dy}{dx}=(2x+5)(2)(x-4)+(x-4)^2(2)$ $=(x-4)(4x+10+2x-8)$ $=(x-4)(6x+2)$ or $6x^2-22x-8$ $\frac{dy}{dt}=-3\frac{dx}{dt}$	M1	Product Rule

	$\frac{dy}{dx}=-3$ $(x-4)(6x+2)=-3$ $6x^2-24x+2x-8+3=0$ $6x^2-22x-5=0$ $x=\frac{-(-22)\pm\sqrt{(-22)^2-4(6)(-5)}}{2(6)}$ $x=\frac{22\pm\sqrt{604}}{12}$ $x=\frac{11+\sqrt{151}}{6}$ or $x=\frac{11-\sqrt{151}}{6}$ (rej since x is positive)	M1 M1 A1	Deduce that $\frac{dy}{dx}=-3$ Solve for x
2a	$\alpha+\beta=7.5$ $\alpha\beta=13$ $\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta$ $=(7.5)^2-2(13)$ $=30.25$ Area $=\frac{1}{2}\alpha\beta$ $=\frac{1}{2}(13)$ $=6.5$ cm^2 Perimeter $=\alpha+\beta+\sqrt{\alpha^2+\beta^2}$ $=7.5+\sqrt{30.25}$ $=13$ cm	M1 M1 A1 A1	Finding both sum and product of roots correctly. Attempt to find $\alpha^2+\beta^2$

2b	Let the roots be α and 2α		
	$\alpha+2\alpha=\frac{-k}{3}$ $3\alpha=\frac{-k}{3}$	M1	Equation from sum of roots
	$2\alpha^2=\frac{96}{3}=32$	M1	Equation from product of roots
	$\alpha^2=\frac{32}{2}$ $\alpha^2=16$		
	$\alpha=4$ or $\alpha=-4$(Rej since the roots are positive) $2\alpha=8$ The 2 roots are 4 and 8	A1	Both roots correct
	$9\alpha=-k$ $k=-36$	A1	
3a	$y=2-\ln(x+1)$ At $x=0$, $y=2-\ln(0+1)$ $y=2$ Coordinates = $(0,2)$	B1	
3b	$y=2-\ln(x+1)$ $\ln(x+1)=2-y$ $e^{2-y}=x+1$	M1	Changing ln to exponential
	$x=e^{2-y}-1$	M1	Function in y

	$\left\|\int_2^3 e^{2-y}-1\,dy\right\|+\int_0^2 e^{2-y}-1\,dy$	M1 M1	Integral of area on left of y-axis Integral of area on the right of y-axis
	$=\left\|\left[-e^{2-y}-y\right]_2^3\right\|+\left[-e^{2-y}-y\right]_0^2$		
	$=\left\|(-e^{2-3}-3)-(-e^0-2)\right\|+\left[(-e^0-2)-(-e^2)\right]$	M1	Integrate exponential function correctly
	$=0.36788+4.38906$ $=4.76$ units^2 (3 s.f.)	M1 A1	Substituting the upper and lower bound correctly
4a	$P(x)=2x^3+ax^2+bx+21$		
	$P\left(\frac{1}{2}\right)=2\left(\frac{1}{2}\right)^3+a\left(\frac{1}{2}\right)^2+b\left(\frac{1}{2}\right)+21=35$	M1	Apply Remainder theorem
	$\frac{1}{4}+\frac{1}{4}a+\frac{1}{2}b+21=35$ $\frac{1}{4}a+\frac{1}{2}b=\frac{55}{4}$ or $a+2b=55$ ---------- (1)		
	$P(-3)=2(-3)^3+a(-3)^2+b(-3)+21=0$ $-54+9a-3b+21=0$ $9a-3b=33$ or $3a-b=11$ ---------- (2) $9a=33+3b$ $a=\frac{33+3b}{9}$ ---------- (3)	M1	Apply Factor theorem
	Sub (3) into (1) $\frac{1}{4}\left(\frac{33+3b}{9}\right)+\frac{1}{2}b=\frac{55}{4}$ $\frac{33+3b}{36}+\frac{1}{2}b=\frac{55}{4}$	M1	Method to solve simultaneous equation

	$33+3b+18b=495$ $21b=462$ $b=22$ $a=11$	A1 A1	
4b	$P(x)=2x^3+11x^2+22x+21$ $P(x)=(x+3)(ax^2+bx+c)$ Comparing coefficient of x^3 term, $a=2$ Comparing constant term, $3c=21$ $c=7$ Comparing coefficient of x term, $3b+c=22$ $3b+7=22$ $b=5$ $P(x)=(x+3)(2x^2+5x+7)$ Let $(x+3)(2x^2+5x+7)=0$ For $2x^2+5x+7=0$, $b^2-4ac=5^2-4(2)(7)$ $=-31$ <0 Hence $2x^2+5x+7=0$ has no real roots $\therefore P(x)=0$ has only one real root The root is $x=-3$	M1 M1 A1 A1	Factorise Use discriminant to check for real roots

5a	$y=\frac{1}{\sqrt{ax+1}}$ $y=(ax+1)^{-\frac{1}{2}}$ $\frac{dy}{dx}=-\frac{1}{2}(ax+1)^{-\frac{3}{2}}(a)$ $=-\frac{a}{2\sqrt{(ax+1)^3}}$ Gradient of tangent at $x=0$ is -1. $\frac{dy}{dx}=-\frac{a}{2\sqrt{1^3}}=-1$ $a=2$	M1 M1 A1	Correct differentiation Equate tangent to -1
5b	$\left(1+\frac{x}{3}\right)^n=1+\binom{n}{1}\left(\frac{x}{3}\right)+\binom{n}{2}\left(\frac{x}{3}\right)^2+\ldots$ $=1+\frac{nx}{3}+\frac{n(n-1)}{2!}\left(\frac{x^2}{9}\right)+\ldots$ $=1+\frac{n}{3}x+\frac{n(n-1)}{18}x^2+\ldots$ $(1-kx)\left(1+\frac{x}{3}\right)^n=(1-kx)\left(1+\frac{n}{3}x+\frac{n(n-1)}{18}x^2+\ldots\right)$ $=1+\frac{n}{3}x-kx+\frac{n(n-1)}{18}x^2-\frac{kn}{3}x^2+\ldots$ $=1+\left(\frac{n}{3}-k\right)x+\left[\frac{n(n-1)}{18}-\frac{kn}{3}\right]x^2+\ldots$	M1 M1 M1	Correct binomial expansion of $\left(1+\frac{x}{3}\right)^n$ for terms up to at least x^2 Correct expansion of $(1-kx)\left(1+\frac{x}{3}\right)^n$ for terms up to at least x^2 Deducing that the coefficient of x term is 0.

	$\frac{n}{3}-k=0$ -------------------- (1) $n=3k$ $\frac{n(n-1)}{18}-\frac{kn}{3}=-\frac{5}{3}$ ---------- (2)	M1	Equating the coefficient of x^2 to $-\frac{5}{3}$
	Sub (1) into (2), $\frac{3k(3k-1)}{18}-\frac{3k^2}{3}=-\frac{5}{3}$ $3k(3k-1)-18k^2=-30$	M1	Solve simultaneous equations
	$9k^2-3k-18k^2+30=0$ $9k^2+3k-30=0$ $3k^2+k-10=0$ $(k+10)(3k-5)=0$ $k=-2$ or $k=\frac{5}{3}$	M1	Correct method to solve quadratic equation
	When $k=-2$, $n=3(-2)=-6$ (Rej) When $k=\frac{5}{3}$, $n=3\left(\frac{5}{3}\right)=5$ $\therefore k=\frac{5}{3}$, $n=5$	A1	Final answer for k and n, with the negative values rejected.

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7i	$x^2-2kx+y^2+2y=-1$ $(x-k)^2-k^2+(y+1)^2-1^2=-1$	M1	Complete the square
	$(x-k)^2+(y+1)^2=k^2$ Since radius is 2 units, $k^2=2^2$ $k=2$	M1 A1	Equating k^2 to 2^2
	Alternative method: $x^2+y^2+2gx+2fy+c=0$ $r=\sqrt{g^2+f^2-c}$ Centre $=(-g,-f)$		
	Conparing coefficient of x and y and constant term. $2f=2$ $f=1$ $c=1$	M1	For finding values of f and c.
	Since radius = 2 $r=\sqrt{g^2+1^2-1}=2$ $g^2=4$ $g=\pm 2$	M1	Equating $\sqrt{g^2+f^2-c}$ to 2.
	If $g=-2$, $k=2$ If $g=2$, $k=-2$ (Rej)	A1	Finding correct value of k, rejecting the negative value

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7ii	Gradient of line through $(3,2)$ and $(0,-1)$ $m_1=\frac{2-(-1)}{3-0}=1$ Let gradient of perpendicular line be m_2 $m_1m_2=-1$ $m_2=-1$ Midpoint between $(3,2)$ and $(0,-1)$ $=\left(\frac{3+0}{2},\frac{2+(-1)}{2}\right)=\left(\frac{3}{2},\frac{1}{2}\right)$ Equation of perpendicular line $=\frac{y-\frac{1}{2}}{x-\frac{3}{2}}=-1$ $y=-x+2$ ----------- (1) $y=2x+2$ ----------- (2) (1) − (2) $x=0,\ y=2$ Centre $C_2=(0,2)$ Radius $C_2=\sqrt{(3-0)^2+(2-2)^2}=3$ Equation C_2: $x^2+(y-2)^2=9$ (Also accept $x^2+y^2-4y-5=0$)	M1 M1 M1 M1 A1	Gradient of perpendicular line Correct Midpoint Attempt to solve for coordinates of centre Correct radius

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	Alternative method Let the centre be $(x,2x+2)$ $(x-3)^2+(2x+2-2)^2=(x-0)^2+(2x+2+1)^2$ $x^2-6x+9+4x^2=x^2+4x^2+12x+9$ $-18x=0$ $x=0$ $y=2(0)+2=2$ Centre of $C_2=(0,2)$ $r=\sqrt{(3-0)^2+(2-2)^2}=3$ Equation of $C_2: x^2+(y-2)^2=9$ (Also accept $x^2+y^2-4y-5=0$	M1 M1 M1 M1 A1	Centre $(x,2x+2)$ Expand and simplify Attempt to solve for coordinates of centre Correct radius
7iii	Distance between two centres $=\sqrt{(2-0)^2+(-1-2)^2}$ $=\sqrt{13}$ Shortest distance $=\sqrt{13}-3$ $=0.606$ units^2 Alternative method Gradient of line between both centres $m=\frac{-1-2}{2-0}=-\frac{3}{2}$ Equation of line between centres	M1 M1 A1 M1	Applying distance formula Show in order to find shortest distance they have to subtract away the radius of C_2 Attempt to find the point on circumference of C_2 which is along the line that joins the centre of C_1 and C_2

	$\frac{y-2}{x-0}=-\frac{3}{2}$ $y=-\frac{3}{2}x+2$ ---------- (1) $x^2+(y-2)^2=3^2$ ---------- (2) Substitute (1) into (2) $x^2+\left(-\frac{3}{2}x\right)^2=3^2$ $\frac{13}{4}x^2=3^2$ $x=\pm\sqrt{\frac{36}{13}}$ $x=\sqrt{\frac{36}{13}}, y=-\frac{3}{2}\sqrt{\frac{36}{13}}+2$		
	Shortest Distance = $\sqrt{\left(2-\sqrt{\frac{36}{13}}\right)^2+\left[-1-\left(-\frac{3}{2}\sqrt{\frac{36}{13}}+2\right)\right]^2}$ $=0.606$ units	M1 A1	Apply distance formula
8i	$\frac{dv}{dt}=a=12t-36$ $\frac{dv}{dt}=0$ $12t-36=0$ $t=3$ s	M1	Correct value of t when v is at its min

	$v=\int 12t-36\,dt$ $v=\frac{12t^2}{2}-36t+c$ Since when $t=0$s, $v=48$m/s $\therefore c=48$ $v=6t^2-36t+48$ Minimum velocity happens at $t=3$ s When $t=3$, $v=6(3)^2-36(3)+48$ $v=-6$ m/s	M1 A1	Correct v function
8ii	At rest, $v=0$ $6t^2-36t+48=0$ $t^2-6t+8=0$ $(t-4)(t-2)=0$ $t=2$ or $t=4$ First at rest at $t=2$ s. $s=\int v\,dt$ $s=\frac{6t^3}{3}-\frac{36t^2}{2}+48t+D$ $s=2t^3-18t^2+48t+D$ $t=0,\ s=0$ $\therefore D=0$ $s=2t^3-18t^2+48t$ At $t=2$ s,	M1 M1	Find the time of instantaneous at rest Integrate to find s function

	$s=2(2)^3-18(2)^2+48(2)$ $s=40\text{m}$	A1	
8iii	$t=4\text{ s},\ s=2(4)^3-18(4)^2+48(4)=32\text{ m}$	M1	Find s at $t=4$
	$t=5\text{ s},\ s=2(5)^3-18(5)^2+48(5)=40\text{m}$	M1	Find s at $t=5$
	Total distance travelled = 40 + 8 + 8 m = 56 m	A1	
8iv	Let $2t^3-18t^2+48t=0$ $2t(t^2-9t+24)=0$ $2t=0$ or $t^2-9t+24=0$	M1	Factorise
	$b^2-4ac=(-9)^2-4(1)(24)$ $=-15$ <0	M1	Conclude $b^2-4ac<0$
	$\therefore$ no real roots for $t^2-9t+24=0$ $s=0$ only at $t=0$, particle never return to its starting point	A1	Conclude no real roots so $s=0$ only at $t=0$
9a	$2^{2x+2}\times5^{x-1}=8^x\times5^{2x}$ $2^{2x+2}\times5^{x-1}=2^{3x}\times5^{2x}$ $\frac{2^{2x+2}}{2^{3x}}=\frac{5^{2x}}{5^{x-1}}$ $2^{2x+2-3x}=5^{2x-(x-1)}$	M1	Combine terms with same base
	$2^{2-x}=5^{x+1}$ $\frac{2^2}{2^x}=5^x\times5$	M1	Split into 2^x and 5^x

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	$\frac{4}{2^x}=5^x\times5$ $10^x=\frac{4}{5}$	A1	
	<u>Alternative Method</u> $2^{2x}(2^2)\times5^x\left(\frac{1}{5}\right)=2^{3x}\times5^{2x}$	M1	Expressing whole expression in base 2 and 5
	$\frac{4}{5}=\frac{2^{3x}\times5^{2x}}{2^{2x}\times5^x}$ $\frac{4}{5}=2^x\times5^x$	M1	Combine terms with same base
	$10^x=\frac{4}{5}$	A1	
9b	$\log_8[\log_4(5x-9)]=\log_{27}3$ $\log_8[\log_4(5x-9)]=\frac{\log_3 3}{\log_3 27}$	M1	Change of base
	$\log_8[\log_4(5x-9)]=\frac{1}{3}$ $8^{\frac{1}{3}}=\log_4(5x-9)$	M1	Change log to index form twice
	$5x-9=4^2$ $x=5$	A1	
9ci	$t=0,\ W=3600(3+e^{-0.18(0)})$ $W=14400$	B1	

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9cii	$t=5$, $W=14400\times 90\%=12960$ $12960=3600\left(3+e^{-0.18k}\right)$ $\frac{18}{5}=3+e^{-0.18(5k)}$ $e^{-0.9k}=\frac{3}{5}$ $-0.9k=\ln\frac{3}{5}$ $k=0.568$ (3 s.f.)	M1 M1 A1	Amount of worms on the farm after 5 days Change to natural log form
9ciii	As $t\to$ large number, $e^{-0.1021644t}\to 0$ $W\to 2800(4+0)=10800$ $\therefore$ The number of worms would not fall below 10000. Alternative method $\frac{W}{3600}=3+e^{-0.18kt}$ $-0.18kt=\ln\left(\frac{W}{3600}-3\right)$ For t to be valid, $\frac{W}{3600}-3>0$ $\frac{W}{3600}>3$ $W>10800$ $\therefore W$ will never be below 10000.	M1 A1	
10a	$\frac{\sin 2A+\cos 2A+1}{\sin 2A+\cos 2A-1}$	M1 M1	Applying $\sin 2A=2\sin A\cos A$

	$=\frac{2\sin A\cos A+2\cos^2 A-1+1}{2\sin A\cos A+1-2\sin^2 A-1}$ $=\frac{2\sin A\cos A+2\cos^2 A}{2\sin A\cos A-2\sin^2 A}$ $=\frac{\sin A\cos A+\cos^2 A}{\sin A\cos A-\sin^2 A}$ $=\frac{\frac{\sin A\cos A}{\sin A\cos A}+\frac{\cos^2 A}{\sin A\cos A}}{\frac{\sin A\cos A}{\sin A\cos A}-\frac{\sin^2 A}{\sin A\cos A}}$ $=\frac{1+\frac{\cos A}{\sin A}}{1-\frac{\sin A}{\cos A}}$ $=\frac{1+\cot A}{1-\tan A}$ (Proved) Alternative Method $\frac{1+\cot A}{1-\tan A}=\frac{\sin A+\cos A}{\sin A}\div\frac{\cos A-\sin A}{\cos A}$ $=\frac{2\cos A(\sin A+\cos A)}{2\sin A(\cos A-\sin A)}$ $=\frac{\sin 2A+2\cos^2 A-1+1}{\sin 2A-2\sin^2 A+1-1}$ $=\frac{\sin 2A+\cos 2A+1}{\sin 2A+\cos 2A-1}$ (proved)	 M1 A1 M1 M1 M1 A1	Applying $\cos 2A=2\cos^2 A-1$ and $\cos 2A=1-2\sin^2 A$ Divide each term by $\sin A\cos A$ $\cot A=\frac{\cos A}{\sin A}$ and $\tan A=\frac{\sin A}{\cos A}$ Applying $\sin 2A=2\sin A\cos A$ Applying $\cos 2A=2\cos^2 A-1$ and $\cos 2A=1-2\sin^2 A$

10bi	$\sin\theta=\frac{AC}{7}\Rightarrow AC=7\sin\theta$ $\cos\theta=\frac{BC}{7}\Rightarrow BC=7\cos\theta$ $\sin(90°-\theta)=\frac{DE}{10}\Rightarrow DE=10\sin(90°-\theta)=10\cos\theta$ $\cos(90°-\theta)=\frac{BE}{10}\Rightarrow BE=10\cos(90°-\theta)=10\sin\theta$ $P=AB+BE+DE+CD+AC$ $=7+10\sin\theta+10\cos\theta+(10-7\cos\theta)+7\sin\theta$ $=7+10\sin\theta+10\cos\theta+10-7\cos\theta+7\sin\theta$ $=17+17\sin\theta+3\cos\theta$	M1 M1 A1	For AC and BC For DE and BE
10bii	$P=17+17\sin\theta+3\cos\theta$ $=17+\sqrt{17^2+3^2}\sin(\theta+0.17467)$ $=17+\sqrt{298}\sin(\theta+0.175)$ Or $=17+17.3\sin(\theta+0.175)$	M1 M1 A1	For R value For α value
10biii	Maximum value of $P=17+\sqrt{289}$ or 34.3 $\sin(\theta+0.17467)=1$ $\theta+0.17467=\frac{\pi}{2}$ $\theta+0.17467=\frac{\pi}{2}$ $\theta=1.40$ (3 s.f.)	B1 B1	

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